HP 12C Platinum

Solutions Handbook

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Introduction

About This Handbook

This *HP 12C Platinum Solutions Handbook* has been designed to supplement the *HP 12C Platinum Owner's Handbook* by providing a variety of applications in the financial area. Programs and/or step-by-step keystroke procedures with corresponding examples in each specific topic are explained. We hope that this book will serve as a reference guide to many of your problems and will show you how to redesign our examples to fit your specific needs.

This book expands the original HP-12C Solutions handbook with additional solutions in algebraic mode. It contains the same RPN program keystrokes and RPN step-by-step procedure keystrokes, in columns headed “12c platinum / 12C RPN Keystrokes”. The alternative algebraic keystrokes are tablulated under “12c platinum ALG Keystrokes”. In program listings the “Display” columns show the keycodes as seen on the HP 12C Platinum.

Appendix A also contains algebraic listings for all the RPN programs given in Part III of the *HP 12C Platinum Owner’s Handbook*.

Presentation of Algebraic and RPN

The conventions used to differentiate between RPN and ALG mode are:

1. **Program Listings**

   Complete and separate listings are given for all programs. They appear side by side in two columns with RPN on the left and Algebraic on the right.

2. **Step-by-Step Keystroke Procedures**

   As for programs separate columns are used, with the RPN keystrokes on the left and the Algebraic keystrokes on the right.

3. **Program Instructions**

   Program instruction steps are generally the same for both modes. Any differences are shown by clearly framing alternative steps and annotating the step or steps in the first frame as RPN, and those in the second as ALG.

4. **Text**

   Occasionally there are small differences which need to be indicated in the text

itself and the ALG alternative is then indicated parenthetically.

5. **Usage of [ENTER]([=])**

   To activate the [=] key it is sufficient just to press [ENTER], with the HP 12C Platinum in ALG mode. In step-by-step and program instructions where the only difference between the modes is that [ENTER] is used in RPN mode and [=] is used in ALG mode, [ENTER]([=]) has been used to indicate both alternatives.

Using the RPN Programs on the HP-12C

Apart from GTO instructions, the keystrokes given in this book are exactly the same for the HP 12C Platinum and the HP-12C. There are two notational differences to bear in mind when typing the RPN programs into the HP-12C:

1. One keycode, for [LSTx], is different.
2. Line numbers tabulated as 000 to 099 refer to lines displayed as 00 to 99 on the HP-12C. The relevant two digit line numbers should be used when typing GTO instructions on the HP-12C.

Notes:

1. All display columns in the examples this book show 2 decimals. This is set by pressing [f]2.
   All programs that do rounding, amortization or depreciation will give slightly different answers if other than 2 decimals are showing.
2. Three of the original programs have been updated:
   a. The last program in the *Real Estate* section (*ATNCPR*) now takes the capital gains tax rate as a separate input, and an extra example has been added showing a different type of tax basis.
   b. In the *Personal Finance* section the IRA program now handles explicit inflation input and withdrawal tax rate input and the Stock Portfolio program handles stock prices with decimal fractions rather than fractions expressed in terms of eighths.
3. Market data (i.e.: interest rates; real estate values, growth rates and rents; taxes; expenses; etc.) used in the examples in this book do not necessarily represent typical current actual data, or reflect recent market trends.

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Real Estate

Refinancing

It can be mutually advantageous to both borrower and lender to refinance an existing mortgage which has an interest rate substantially below the current market rate, with a loan at a below-market rate. The borrower has the immediate use of tax-free cash, while the lender has substantially increased debt service on a relatively small cash outlay.

To find the benefits to both borrower and lender:

1. Calculate the monthly payment on the existing mortgage.
2. Calculate the monthly payment on the new mortgage.
3. Calculate the net monthly payment received by the lender (and paid by the borrower) by adding the figure found in Step 1 to the figure found in Step 2.
4. Calculate the Net Present Value (NPV) to the lender of the net cash advanced.
5. Calculate the yield to the lender as an IRR.
6. Calculate the NPV to the borrower of the net cash received.

Example: An investment property has an existing mortgage which originated 8 years ago with an original term of 25 years, fully amortized in level monthly payments at 6.5% interest. The current balance is $133,190.

Although the going current market interest rate is 11.5%, the lender has agreed to refinance the property with a $200,000, 17 year, level-monthly-payment loan at 9.5% interest.

What are the *NPV* and effective yield to the lender on the net amount of cash actually advanced?

What is the *NPV* to the borrower on this amount if he can earn a 15.25% equity yield rate on the net proceeds of the loan?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
g END f CLEAR FIN 17 g 12x 6.5 g 12÷ 133190 PV PMT STO 0	g END f CLEAR FIN 17 g 12x 6.5 g 12÷ 133190 PV PMT STO 0	**-1,080.33**	Monthly payment on existing mortgage received by lender.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
9.5 g 12÷ 200000 CHS PV PMT	9.5 g 12÷ 200000 CHS PV PMT	**1,979.56**	Monthly payment on new mortgage.
RCL 0 + PMT	+ RCL 0 PMT	**899.23**	Net monthly payment (to lender).
RCL PV 133190 + STO 0	RCL PV + 133190 = STO 0	**-66,810.00**	Net amount of cash advanced (by lender).
11.5 g 12÷ PV	11.5 g 12÷ PV PV	**-80,425.02**	Present value of net monthly payment.
RCL 0 −	− RCL 0 =	**-13,615.02**	*NPV* to lender of net cash advanced.
RCL 0 PV i 12 X	RCL 0 PV i X 12 =	**14.83**	% nominal yield (*IRR*).
15.25 g 12÷ PV	15.25 g 12÷ PV PV	**-65,376.72**	Present value of net monthly payment at 15.25%.
RCL 0 −	− RCL 0 =	**1,433.28**	*NPV* to borrower.

Wrap-Around Mortgage

A wrap-around mortgage is essentially the same as a refinancing mortgage, except that the new mortgage is granted by a different lender, who assumes the payments on the existing mortgage, which remains in full force. The new (second) mortgage is thus "wrapped around" the existing mortgage. The "wrap-around" lender advances the net difference between the new (second) mortgage and the existing mortgage in cash to the borrower, and receives as net cash flow, the difference between debt service on the new (second) mortgage and debt service on the existing mortgage.

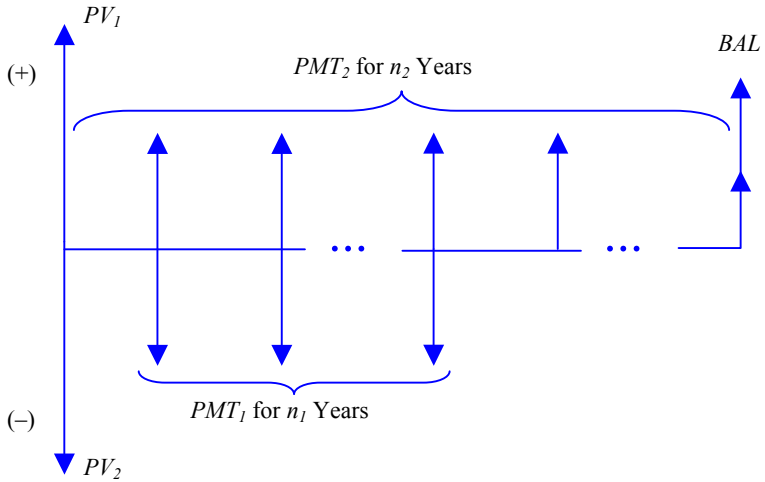
When the terms of the original mortgage and the wrap-around are the same, the procedures in calculating NPV and IRR to the lender and NPV to the borrower are exactly the same as those presented in the preceding section on refinancing.

Example 1: A mortgage loan on an income property has a remaining balance of $200,132.06. When the load originated 8 years ago, it had a 20 year term with full amortization in level monthly payments at 6.75% interest.

A lender has agreed to "wrap" a $300,000 second mortgage at 10%, with full amortization in level monthly payments over 12 years. What is the effective yield (IRR) to the lender on the net cash advanced?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
g END f CLEAR FIN 20 ENTER 8 − g 12X	g END f CLEAR FIN 20 − 8 = g 12X	**144.00**	Total number of months remaining in original loan (into n).
6.75 g 12÷	6.75 g 12÷	**0.56**	Monthly interest rate (into i).
200132.06 PV	200132.06 PV	**200,132.06**	Loan amount (into PV).
PMT STO 0	PMT STO 0	**-2,031.55**	Monthly payment on existing mortgage (calculated).
10 g 12÷	10 g 12÷	**0.83**	Monthly interest on wrap-around.
300000 CHS PV	300000 CHS PV	**-300,000.00**	Amount of wrap-around (into PV).
PMT	PMT	**3,585.23**	Monthly payment on wrap-around (calculated).
RCL 0 + PMT	+ RCL 0 PMT	**1,553.69**	Net monthly payment received (into PMT).
RCL PV 200132.06 + PV	RCL PV + 200132.06 PV	**-99,867.94**	Net cash advanced (into PV).
i 12 X	i X 12 =	**15.85**	Nominal yield (IRR) to lender (calculated).

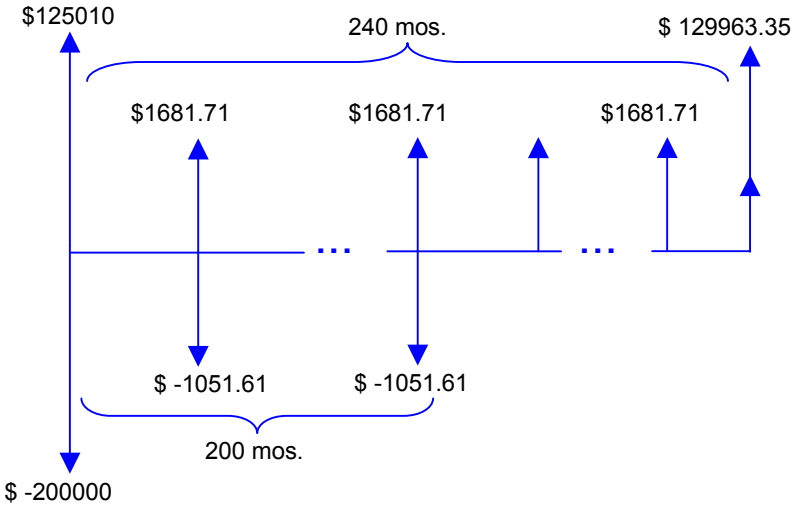
Sometimes the wrap around mortgage will have a longer payback period than the original mortgage, or a balloon payment may exist.



Where:

n_1 = number of years remaining in original mortgage
PMT_1 = yearly payment of original mortgage
PV_1 = remaining balance of original mortgage
n_2 = number of years in wrap-around mortgage
PMT_2 = yearly payment of wrap-around mortgage
PV_2 = total amount of wrap-around mortgage
BAL = balloon payment

Example 2: A customer has an existing mortgage with a balance of $125,010, a remaining term of 200 months, and a $1051.61 monthly payment. He wishes to obtain a $200,000, 9 ½% wrap-around with 240 monthly payments of $1681.71 and a balloon payment at the end of the 240th month of $129,963.35. If you, as a lender, accept the proposal, what is your rate of return?



12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
g END f CLEAR FIN 200000 CHS ENTER 125010 + g CFo	g END f CLEAR FIN 200000 CHS + 125010 g CFo	**-74,990.00**	Net investment.
1051.61 CHS ENTER 1681.71 +	1051.61 CHS + 1681.71 g CFj	**630.10**	Net cash flow received by lender.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
g CFj 99 g Nj x≷y g CFj x≷y g Nj x≷y g CFj 2 g Nj	99 g Nj x≷y g CFj x≷y g Nj x≷y g CFj 2 g Nj		The above cash flow occurs 200 times.
g LSTx g CFj	1681.71 g CFj	**1,681.71**	Next cash flow received by lender.
39 g Nj	39 g Nj	**39.00**	Cash flow occurs 39 times.
x≷y 129963.35 + g CFj	x≷y + 129963.35 g CFj	**131,645.06**	Final cash flow.
f IRR 12 X	f IRR X 12 =	**11.84**	Rate of return to lender.

If you, as a lender, know the yield on the entire transaction, and you wish to obtain the payment amount on the wrap-around mortgage to achieve this yield, use the following procedure. Once the monthly payment is known, the borrower's periodic interest rate may also be determined.

1. Press the g END and press f CLEAR FIN.
2. Key in the remaining periods of the original mortgage and press n.
3. Key in the desired annual yield and press g 12÷.
4. Key in the monthly payment to be made by the lender on the original mortgage and press CHS PMT.
5. Press PV.
6. **RPN:** Key in the net amount of cash advanced and press + CHS PV.
6. **ALG:** Press +, key in the net amount of cash advanced and press = CHS PV.
7. Key in the total term of the wrap-around mortgage and press n.
8. If a balloon payment exists, key it in and press FV.
9. Press PMT to obtain the payment amount necessary to achieve the desired yield.
10. Key in the amount of the wrap-around mortgage and press CHS PV i to obtain the borrower's periodic interest rate.

Example 3: Your firm has determined that the yield on a wrap-around mortgage should be 12% annually. In the previous example, what monthly payment must be received to achieve this yield on a $200,000 wrap-around? What interest rate is the borrower paying?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
g END f CLEAR FIN 200 n 12 g 12÷	g END f CLEAR FIN 200 n 12 g 12÷		Number of periods and monthly interest rate.
1051.61 CHS PMT	1051.61 CHS PMT		Monthly payment.
PV 74990 + CHS PV	PV + 74990 = CHS PV	**-165,776.92**	Present value of payments plus cash advanced.
240 n 129963.35 FV PMT	240 n 129963.35 FV PMT	**1,693.97**	Monthly payment received by lender.
200000 CHS PV i 12 X	200000 CHS PV i X 12 =	**9.58**	Annual interest rate paid by borrower.

Income Property Cash Flow Analysis

Before-Tax Cash Flows

The before-tax cash flows applicable to real estate analysis and problems are:

- Potential Gross Income
- Effective Gross Income
- Net Operating Income (also called Net Income Before Recapture)
- Cash Throw-off to Equity (also called Gross Spendable Cash)

The derivation of these cash flows follows a set sequence:

1. Calculate Potential Gross Income by multiplying the rent per unit times the number of units, times the number of rental payment periods per year. This gives the rental income the property would generate if it were fully occupied.
2. Deduct Allowance for Vacancy and Rental Loss. This is usually expressed as a percentage. The result is Rent Collections (which is also Effective Gross Income if there is no "Other Income").
3. Add "Other Income" such as receipts from concessions (laundry equipment, etc.), produced from sources other than the rental office space. This is Effective Gross Income.
4. Deduct Operating Expenses. These are expenditures the landlord-investor must make, by contract or custom, to preserve the property and keep in capable of producing the gross income. The result is the Net Operating Income.
5. Deduct Annual Debt Service on the mortgage. This produces Cash Throw-Off to Equity.

Thus:

Effective Gross Income =Potential Gross Income - Vacancy Loss + Other Income.

Net Operating Income =Effective Gross Income - Operating Expenses.

Cash Throw-Off =Net Operating Income - Annual Dept Service.

Example: A 60 unit apartment building has rentals of $250 per unit per month. With a 5% vacancy rate, the annual operating cost is $76,855.

The property has just been financed with a $700,000 mortgage, fully amortized in a level monthly payments at 11.5% over 20 years.

a. What is the Effective Gross Income?

b. What is the Net Operating Income?

c. What is the Cash Throw-Off to Equity?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
g END f CLEAR FIN 60 ENTER 250 X 12 X	g END f CLEAR FIN 60 X 250 X 12 −	**180,000.00**	Potential Gross Income.
5 % −	5 % −	**171,000.00**	Effective Gross Income.
76855 −	76855 =	**94,145.00**	Net Operating Income.
20 g 12x 11.5 g 12÷ 700000 PV PMT 12 X	20 g 12x R↓ 11.5 g 12÷ R↓ 700000 PV PMT X 12 +	**-89,580.09**	Annual Debt Service.
+	x≷y =	**4,564.91**	Cash Throw-Off.

Before-Tax Reversions (Resale Proceeds)

The reversion receivable at the end of the income projection period is usually based on forecast or anticipated resale of the property at that time. The before tax reversion amount applicable to real estate analysis and problems are:

- Sale Price.
- Cash Proceeds of Resale.
- Outstanding Mortgage Balance.
- Net Cash Proceeds of Resale to Equity.

The derivation of these reversions is as follows:

1. Forecast or estimate Sales Price. Deduct sales and Transaction Costs. The result is the Proceeds of Resale.

2. Calculate the Outstanding Balance of the Mortgage at the end of the Income Projection Period and subtract it from Proceeds of Resale. The result is Net Cash Proceeds of Resale.

Thus:

Cash Proceeds of Resale = Sales Price - Transaction Costs.

Net Cash Proceeds of Resale = Cash Proceeds of Resale - Outstanding Mortgage Balance.

Example: The apartment property in the preceding example is expected to be resold in 10 years. The anticipated resale price is $800,000. The transaction costs are expected to be 7% of the resale price. The mortgage is the same as that indicated in the preceding example.

- What will the Mortgage Balance be in 10 years?
- What are the Cash Proceeds of Resale and Net Cash Proceeds of Resale?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
g END f CLEAR FIN	g END f CLEAR FIN		
20 g 12x	20 g 12x	**240.00**	Mortgage term.
11.5 g 12÷	11.5 g 12÷	**0.96**	Mortgage rate.
700000 PV	700000 PV		Property value.
PMT	PMT	**-7,465.01**	Monthly payment.
10 g 12x	10 g 12x	**120.00**	Projection period.
FV	FV FV	**-530,956.57**	Mortgage balance in 10 years.
800000 ENTER	800000 −		Estimated resale.
7 % −	7 % +	**744,000.00**	Cash Proceeds of Resale.
+	x≷y =	**213,043.43**	Net Cash Proceeds of Resale.

After-Tax Cash Flows

The After-Tax Cash Flow (ATCF) is found for the each year by deducting the Income Tax Liability for that year from the Cash Throw Off.

Where Taxable Income = Net Operating Income - interest - depreciation,

Tax Liability = Taxable Income x Marginal Tax Rate,

and After Tax Cash Flow = Cash Throw Off - Tax Liability.

The After-Tax Cash Flow for the initial and successive years may be calculated by the following HP 12C Platinum program. This program calculates the Net Operating Income using the Potential Gross Income, operational cost and vacancy rate. The Net Operating

income is readjusted each year from the growth rates in Potential Gross Income and operational costs.

The user is able to change the method of finding the depreciation from declining balance to straight line. To make the change, key in [f] [SL] at line 032 (ALG: 026) of the program in place of [f] [DB].

12c platinum / 12C RPN KEYSTROKES	DISPLAY
[f] [P/R]	
[f] CLEAR [PRGM]	000,
0	001, 0
[n]	002, 11
[STO] 1	003, 44 1
[RCL] 7	004, 45 7
[EEX]	005, 26
2	006, 2
[÷]	007, 10
[STO] 7	008, 44 7
1	009, 1
[STO] [+] 1	010,44 40 1
1	011, 1
2	012, 2
[f] [AMORT]	013, 42 11
[STO] 0	014, 44 0
[RCL] 5	015, 45 5
[n]	016, 11
[RCL] [i]	017, 45 12
[RCL] 6	018, 45 6
[i]	019, 12
[R↓]	020, 33
[STO] 6	021, 44 6
[R↓]	022, 33
[RCL] [PV]	023, 45 13
[RCL] 4	024, 45 4
[PV]	025, 13
[R↓]	026, 33
[STO] 4	027, 44 4
[R↓]	028, 33
[g] [x=0]	029, 43 35
[g] [GTO] 036	030,43,33,036
[RCL] 1	031, 45 1
[f] [DB]	032, 42 25
[STO] [−] 0	033,44 30 0
0	034, 0
[g] [GTO] 017	035,43,33,017

12c platinum ALG KEYSTROKES	DISPLAY
[f] [P/R]	
[f] CLEAR [PRGM]	000,
0	001, 0
[n]	002, 11
1	003, 1
2	004, 2
[f] [AMORT]	005, 42 11
[STO] 0	006, 44 0
[RCL] 5	007, 45 5
[n]	008, 11
[RCL] [i]	009, 45 12
[RCL] 6	010, 45 6
[i]	011, 12
[R↓]	012, 33
[STO] 6	013, 44 6
[R↓]	014, 33
[RCL] [PV]	015, 45 13
[RCL] 4	016, 45 4
[PV]	017, 13
[R↓]	018, 33
[STO] 4	019, 44 4
[R↓]	020, 33
[g] [x=0]	021, 43 35
[g] [GTO] 030	022,43,33,030
1	023, 1
[STO] [+] 1	024,44 40 1
[RCL] 1	025, 45 1
[f] [DB]	026, 42 25
[STO] [−] 0	027,44 30 0
0	028, 0
[g] [GTO] 009	029,43,33,009
[RCL] 2	030, 45 2
[−]	031, 30
[RCL] [•] 0	032,45 48 0
[%]	033, 25
[−]	034, 30
[RCL] 3	035, 45 3

12c platinum / 12C RPN KEYSTROKES	DISPLAY
[n]	036, 11
[RCL] 2	037, 45 2
[RCL] 8	038, 45 8
[%]	039, 25
[STO] [+] 2	040,44 40 2
[R↓]	041, 33
[RCL] [•] 0	042,45 48 0
[%]	043, 25
[−]	044, 30
[RCL] 3	045, 45 3
[RCL] 9	046, 45 9
[%]	047, 25
[STO] [+] 3	048,44 40 3
[R↓]	049, 33
[−]	050, 30
1	051, 1
[RCL] 7	052, 45 7
[STO] [X] 0	053,44 20 0
[−]	054, 30
[X]	055, 20
[RCL] [PMT]	056, 45 14
1	057, 1
2	058, 2
[X]	059, 20
[+]	060, 40
[RCL] 0	061, 45 0
[−]	062, 30
[RCL] 1	063, 45 1
[g] [PSE]	064, 43 31
[x≷y]	065, 34
[R/S]	066, 31
[g] [GTO] 009	067,43,33,009
[f] [P/R]	

12c platinum ALG KEYSTROKES	DISPLAY
[=]	036, 36
[STO] [+] 0	037,44 40 0
[+]	038, 40
[RCL] [PMT]	039, 45 14
[g] [12X]	040, 43 11
[=]	041, 36
[RCL] 7	042, 45 7
[%]	043, 25
[STO] [X] 0	044,44 20 0
[R↓]	045, 33
[−]	046, 30
[RCL] 0	047, 45 0
[=]	048, 36
[RCL] 1	049, 45 1
[g] [PSE]	050, 43 31
[x≷y]	051, 34
[R/S]	052, 31
[RCL] 2	053, 45 2
[+]	054, 40
[RCL] 8	055, 45 8
[%]	056, 25
[=]	057, 36
[STO] 2	058, 44 2
[RCL] 3	059, 45 3
[+]	060, 40
[RCL] 9	061, 45 9
[%]	062, 25
[=]	063, 36
[STO] 3	064, 44 3
[g] [GTO] 001	065,43,33,001
[f] [P/R]	

	REGISTERS		
n: Used	i: Annual %	PV: Used	PMT: Monthly
FV: 0	R_0: Used	R_1: Counter	R_2: *PGI*
R_3: Oper. cost	R_4: Dep. value	R_5: Dep. Life	R_6: Factor (DB)
R_7: Tax Rate	R_8: % gr. (PGI)	R_9: % gr. (op)	$R_{.0}$: Vacancy rt.

Program Instructions:

1. Press [g] [END] and press [f] CLEAR [REG].
2. Key in loan values:
   - Key in annual interest rate and press [g] [12÷]
   - Key in principal to be paid and press [PV]
   - Key in monthly payment and press [CHS] [PMT]

   (If any of the values are not known, they should be solved for.)
3. Key in Potential Gross Income (*PGI*) and press [STO] 2.
4. Key in Operational cost and press [STO] 3.
5. Key in depreciable value and press [STO] 4.
6. Key in depreciable life and press [STO] 5.
7. Key in factor (for declining balance only) and press [STO] 6.
8. Key in the Marginal Tax Rate (as a percentage) and press [STO] 7.
9. Key in the growth rate in Potential Gross Income (0 for no growth) and press [STO] 8.
10. Key in the growth rate in operational cost (0 if no growth) and press [STO] 9.
11. Key in the vacancy rate (0 for no vacancy rate) and press [STO] [•] 0.
12. **RPN:** Key in the desired depreciation function at line 032 in the program.

12. **ALG:** Key in the desired depreciation function at line 026 in the program.

13. Press [R/S] to compute *ATCF*. The display will pause showing the year and then will stop with the *ATCF* for that year. The Y-register contains the year.
14. Continue pressing [R/S] to compute successive After-Tax Cash Flows.

Example 1: A triplex was recently purchased for $100,000 with a 30 year loan at 12.25% and a 20% down payment. Not including a 5% annual vacancy rate, the potential gross income is $9,900 with an annual growth rate of 6%. Operating expenses are $3,291.75 with a 2.5% growth rate. The depreciable value is $75,000 with a projected useful life of $20 years. Assuming a 125% declining balance depreciation, what are the After-Tax Cash Flows for the first 10 years if the investors Marginal Tax Rate is 35%?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
[g] [END] [f] CLEAR [REG] 100000 [ENTER]	[g] [END] [f] CLEAR [REG] 100000 [−]		
20 [%] [−] [PV]	20 [%] [PV]	**80,000.00**	Mortgage amount.
12.25 [g] [12÷]	12.25 [g] [12÷]	**1.02**	Monthly interest rate.
30 [g] [12×]	30 [g] [12×]	**360**	Mortgage term.
[PMT]	[PMT] [PMT]	**-838.32**	Monthly payment.
9900 [STO] 2	9900 [STO] 2	**9,900.00**	Potential Gross Income.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
3291.75 STO 3	3291.75 STO 3	**3,291.75**	1st year operating cost.
75000 STO 4	75000 STO 4	**75,000.00**	Depreciable value.
20 STO 5	20 STO 5	**20.00**	Useful life.
125 STO 6	125 STO 6	**125.00**	Declining balance factor.
35 STO 7	35 STO 7	**35.00**	Marginal Tax Rate.
6 STO 8	6 STO 8	**6.00**	Potential Gross Income growth rate.
2.5 STO 9	2.5 STO 9	**2.50**	Operating cost growth.
5 STO • 0	5 STO • 0	**5.00**	Vacancy rate.
R/S	R/S	**1.00** **-1,020.88**	Year 1 $ATCF_1$
R/S	R/S	**2.00** **-822.59**	Year 2 $ATCF_2$
R/S	R/S	**3.00** **-598.85**	Year 3 $ATCF_3$
R/S	R/S	**4.00** **-348.94**	Year 4 $ATCF_4$
R/S	R/S	**5.00** **-72.16**	Year 5 $ATCF_5$
R/S	R/S	**6.00** **232.35**	Year 6 $ATCF_6$
R/S	R/S	**7.00** **565.48**	Year 7 $ATCF_7$
R/S	R/S	**8.00** **928.23**	Year 8 $ATCF_8$
R/S	R/S	**9.00** **1,321.62**	Year 9 $ATCF_9$
R/S	R/S	**10.00** **1,746.81**	Year 10 $ATCF_{10}$

Example 2: An office building was purchased for $1,400,000. The value of depreciable improvements is $1,200,000 with a 35 year economic life. Straight line depreciation will be used. The property is financed with a $1,050,000 loan. The terms of the loan are 9.5% interest and $9,173.81 monthly payments for 25 years. The office building generates a Potential Gross Income of $175,200 which grows at a 3.5% annual rate. The operating cost is $40,296.00 with a 1.6% annual growth rate. Assuming a Marginal Tax Rate of 50% and a vacancy rate of 7%, what are the After-Tax Cash Flows for the first 5 years?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
g END f CLEAR REG 1050000 PV 9173.81 CHS PMT 9.5 g 12÷ 25 g 12x 175200 STO 2	g END f CLEAR REG 1050000 PV 9173.81 CHS PMT 9.5 g 12÷ 25 g 12x 175200 STO 2	**175,200.00**	Potential Gross Income.
40296 STO 3	40296 STO 3	**40,296.00**	1st year operating cost.
1200000 STO 4	1200000 STO 4	**1,200,000.00**	Depreciable value.
35 STO 5	35 STO 5	**35.00**	Depreciable life.
50 STO 7	50 STO 7	**50.00**	Marginal tax rate.
3.5 STO 8	3.5 STO 8	**3.50**	Potential Gross Income growth rate.
1.6 STO 9	1.6 STO 9	**1.60**	Operating cost growth rate.
7 STO • 0	7 STO • 0	**7.00**	Vacancy rate.
g GTO 031	g GTO 025	**7.00**	Go to dep. step.
f P/R f SL	f P/R f SL	**032, 42 23** **026, 42 23**	**RPN:**Change to SL **ALG:**depreciation
f P/R R/S	f P/R R/S	**1.00** **18,021.07**	Year 1 $ATCF_1$
R/S	R/S	**2.00** **20,014.26**	Year 2 $ATCF_2$
R/S	R/S	**3.00** **22,048.90**	Year 3 $ATCF_3$
R/S	R/S	**4.00** **24,123.14**	Year 4 $ATCF_4$
R/S	R/S	**5.00** **26,234.69**	Year 5 $ATCF_5$

After-Tax Net Cash Proceeds of Resale

The After-Tax Net Cash Proceeds of Resale (*ATNCPR*) is the after-tax reversion to equity; generally, the estimated resale price of the property less commissions, outstanding debt and any tax claim.

The After-Tax Net Cash Proceeds can be found using the HP 12C Platinum program which follows.

20 Real Estate

This program uses declining balance depreciation to find the amount of depreciation from purchase to sale. This amount is used to determine the excess depreciation (which is equal to the amount of actual depreciation minus the amount of the straight line depreciation).

The Marginal Tax Rate (MTR) that the user inputs is applied to this excess depreciation.

The Capital Gains Tax Rate (CGTR) that the user inputs is applied to the capital gain from purchase to sale less the expenses of sale (i.e. the *NCPR* or Net cash Proceeds of Resale), *plus* the straight line depreciation.

The user may change to a different depreciation method by keying in the desired function at line 026 (ALG: 029) in place of [f] [DB].

In addition the user may nullify the straight line depreciation by keying in a 0 at line 035 (ALG: 039) in place of [f] [SL]. This means that all of the actual depreciation from purchase to sale is then treated as "excess" or unrecaptured depreciation. This is illustrated below in *Example 2*.

12c platinum / 12C RPN KEYSTROKES	DISPLAY	12c platinum ALG KEYSTROKES	DISPLAY
[f] [P/R]		[f] [P/R]	
[f] CLEAR [PRGM]	000,	[f] CLEAR [PRGM]	000,
[g] [END]	001, 43 8	[g] [END]	001, 43 8
[STO] 2	002, 44 2	[STO] 2	002, 44 2
[g] [12x]	003, 43 11	[g] [12x]	003, 43 11
[R↓]	004, 33	[R↓]	004, 33
[%]	005, 25	[x≷y]	005, 34
[−]	006, 30	[−]	006, 30
[STO] 0	007, 44 0	[x≷y]	007, 34
[x≷y]	008, 34	[%]	008, 25
[−]	009, 30	[−]	009, 30
[RCL] 7	010, 45 7	[STO] 0	010, 44 0
[%]	011, 25	[x≷y]	011, 34
[STO] 1	012, 44 1	[x]	012, 20
[RCL] [PMT]	013, 45 14	[RCL] 7	013, 45 7
[f] [RND]	014, 42 14	[=]	014, 36
[PMT]	015, 14	[STO] 1	015, 44 1
[FV]	016, 15	[RCL] [PMT]	016, 45 14
[STO] [+] 0	017,44 40 0	[f] [RND]	017, 42 14
[f] CLEAR [FIN]	018, 42 34	[PMT]	018, 14
[RCL] 3	019, 45 3	[FV]	019, 15
[PV]	020, 13	[STO] [+] 0	020,44 40 0
[RCL] 4	021, 45 4	[f] CLEAR [FIN]	021, 42 34
[n]	022, 11	[RCL] 3	022, 45 3
[RCL] 5	023, 45 5	[PV]	023, 13
[i]	024, 12	[RCL] 4	024, 45 4
[RCL] 2	025, 45 2	[n]	025, 11
[f] [DB]	026, 42 25	[RCL] 5	026, 45 5
[R↓]	027, 33	[i]	027, 12

12c platinum / 12C RPN KEYSTROKES	DISPLAY
RCL 3	028, 45 3
x≷y	029, 34
−	030, 30
RCL 6	031, 45 6
%	032, 25
STO + 1	033,44 40 1
RCL 2	034, 45 2
f SL	035, 42 23
RCL 2	036, 45 2
X	037, 20
RCL 7	038, 45 7
RCL 6	039, 45 6
−	040, 30
%	041, 25
STO + 1	042,44 40 1
RCL 0	043, 45 0
RCL 1	044, 45 1
−	045, 30
g GTO 000	046,43,33,000
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
RCL 2	028, 45 2
f DB	029, 42 25
x≷y	030, 34
RCL 3	031, 45 3
−	032, 30
x≷y	033, 34
X	034, 20
RCL 6	035, 45 6
=	036, 36
STO + 1	037,44 40 1
RCL 2	038, 45 2
f SL	039, 42 23
X	040, 20
RCL 2	041, 45 2
=	042, 36
RCL 7	043, 45 7
−	044, 30
RCL 6	045, 45 6
X	046, 20
x≷y	047, 34
+	048, 40
RCL 1	049, 45 1
X	050, 20
1	051, 1
%	052, 25
−	053, 30
STO 1	054, 44 1
RCL 0	055, 45 0
x≷y	056, 34
=	057, 36
g GTO 000	058,43,33,000
f P/R	

	REGISTERS		
n: Used	i: Used	PV: Used	PMT: Used
FV: Used	R_0: *NCPR*	R_1: Tax paid	R_2: Desired yr.
R_3: Dep. value	R_4: Dep. life	R_5: Factor	R_6: MTR
R_7: CGTR	R_8-$R_{.3}$: Unused		

Program Instructions:

1. Key in the program and press f CLEAR REG.

2. Key in the loan values:
   - Key in annual interest rate and press [g] [12÷].
   - Key in mortgage amount and press [PV].
   - Key in monthly payment and press [CHS] [PMT].

   (If any of the values are unknown, they should be solved for and if one has to be solved for then the correct payment mode needs to be set)
3. Key in depreciable value and press [STO] 3.
4. Key in depreciable life in years and press [STO] 4.
5. Key in accelerated depreciation factor for the declining balance method and press [STO] 5.
6. Key in your Marginal Tax Rate as a percentage and press [STO] 6.
7. Key in the Capital Gains Tax Rate as a percentage and press [STO] 7.
8. Key in the purchase price and press [ENTER] ([=]).
9. Key in the sale price and press [ENTER] ([=]).
10. Key in the % commission charged on the sale and press [ENTER] ([=]).

RPN: If a dollar value is desired instead of a commission rate, key in [g] [END], which does not affect the register values, at line 005 of the program.
ALG: If a dollar value is desired instead of a commission rate, key in [g] [END], which does not affect the register values, at line 008 of the program.

11. Key in the number of years after purchase and press [R/S]. The *ATNCPR* is displayed.
12. To see the *NCPR* press [RCL] 0 and to see the tax due press [RCL] 1.

Example 1: An apartment complex, purchased for $900,000 ten years ago, is sold for $1,750,000. The closing cost is 8% of the sale price, the income tax rate is 48% and the capital gains tax rate is 19.2%.

A $700,000 loan for 20 years at 9.5% annual interest was used to purchase the complex. When it was purchased the depreciable value was $750,000 with a useful life of 25 years. Using 125% declining balance depreciation, what are the After-Tax Net Cash Proceeds in year 10?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
[g] [END] [f] CLEAR [REG] 700000 [PV]	[g] [END] [f] CLEAR [REG] 700000 [PV]	**0.00** **700,000.00**	Mortgage.
9.5 [g] [12÷]	9.5 [g] [12÷]	**0.79**	Monthly interest.
20 [g] [12x]	20 [g] [12x]	**240.00**	Number of payments.
[PMT]	[PMT] [PMT]	**-6,524.92**	Monthly payment.
750000 [STO] 3	750000 [STO] 3	**750,000.00**	Depreciable value.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
25 STO 4	25 STO 4	**25.00**	Depreciable life.
125 STO 5	125 STO 5	**125.00**	Factor.
48 STO 6	48 STO 6	**48.00**	Marginal Tax Rate.
19.2 STO 7	19.2 STO 7	**19.20**	Capital Gains Tax Rate.
900000 ENTER	900000 =	**900,000.00**	Purchase price.
1750000 ENTER	1750000 =	**1,750,000.00**	Sale price.
8 ENTER	8 =	**8.00**	Commission rate.
10 R/S	10 R/S	**911,372.04**	*ATNCPR.*
RCL 0	RCL 0	**1,105,746.74**	*NCPR.*
RCL 1	RCL 1	**194,374.70**	Tax due on resale.

Example 2: Now, re-do the previous example assuming all depreciation is treated as excess or unrecaptured depreciation, with MTR=25% and CGTR=15%.

First the f SL in the program must be replaced with 0. This may be done as follows:

RPN: Press g GTO 034 f P/R 0 f P/R.
ALG: Press g GTO 038 f P/R 0 f P/R.

The data stored in registers R_3-R_5 need not be re-entered.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
f CLEAR FIN	f CLEAR FIN	**0.00**	
700000 PV	700000 PV	**700,000.00**	Mortgage.
9.5 g 12÷	9.5 g 12÷	**0.79**	Monthly interest.
20 g 12x	20 g 12x	**240.00**	Number of payments.
PMT	PMT PMT	**-6,524.92**	Monthly payment.
25 STO 6	25 STO 6	**25.00**	Marginal Tax Rate.
15 STO 7	15 STO 7	**15.00**	Capital Gains Tax Rate.
900000 ENTER	900000 =	**900,000.00**	Purchase price.
1750000 ENTER	1750000 =	**1,750,000.00**	Sale price.
8 ENTER	8 =	**8.00**	Commission rate.
10 R/S	10 R/S	**924,009.92**	*ATNCPR.*
RCL 0	RCL 0	**1,105,746.74**	*NCPR.*
RCL 1	RCL 1	**181,736.83**	Tax due on resale.

Lending

Loan With a Constant Amount Paid Towards Principal

This type of loan is structured such that the principal is repaid in equal installments with the interest paid in addition. Therefore each periodic payment has a constant amount applied toward the principle and a varying amount of interest.

Loan Reduction Schedule

If the constant periodic payment to principal, annual interest rate, and loan amount are known, the total payment, interest portion of each payment, and remaining balance after each successive payment may be calculated as follows:

RPN Mode:

1. Key in the constant periodic payment to principal and press [STO] 0.
2. Key in periodic interest rate and press [ENTER] [ENTER] [ENTER].
3. Key in the loan amount. If you wish to skip to another time period, press [ENTER].
   Then key in the number of payments to be skipped, and press [RCL] 0 [X] [−].
4. Press [x≷y] [%] to obtain the interest portion of the payment.
5. Press [RCL] 0 [+] to obtain the total payment.
6. Press [CLx] [RCL] 0 [−] to obtain the remaining balance of the loan.
7. Return to step 4 for each successive payment.

ALG Mode:

1. Key in the constant periodic payment to principal and press [STO] 0.
2. Key in the loan amount and press [STO] 1.
3. Key in periodic interest rate and press [STO] 2.
   If you wish to skip to another time period, key in the number of payments to be skipped, and press [X] [RCL] 0 [=] [STO] [−] 1.
4. Press [RCL] 1 [X] [RCL] 2 [%] [+] to obtain the interest portion of the payment.
5. Press [RCL] 0 [STO] [−] 1 [=] to obtain the total payment.
6. Press [RCL] 1 to obtain the remaining balance of the loan.
7. Return to step 4 for each successive payment.

Example 1: A $60,000 land loan at 10% interest calls for equal semi-annual principal payments over a 6-year maturity. What is the loan reduction schedule for the first year? (Constant payment to principal is $5000 semi-annually). What is the fourth year's schedule (skip 4 payments)?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
5000 STO 0 10 ENTER 2 ÷ ENTER ENTER ENTER	5000 STO 0 60000 STO 1 10 ÷ 2 = STO 2	**5.00**	Semi-annual interest rate.
60000 x≷y %	RCL 1 X RCL 2 % +	**3,000.00**	First payment's interest.
RCL 0 +	RCL 0 STO − 1 =	**8,000.00**	Total first payment.
CLx RCL 0 −	RCL 1	**55,000.00**	Remaining balance.
x≷y %	X RCL 2 % +	**2,750.00**	Second payment's interest.
RCL 0 +	RCL 0 STO − 1 =	**7,750.00**	Total second payment.
CLx RCL 0 −	RCL 1	**50,000.00**	Remaining balance after the first year.
4 RCL 0 X − x≷y %	4 X RCL 0 = STO − 1 RCL 1 X RCL 2 % +	**1,500.00**	Seventh payment's interest.
RCL 0 +	RCL 0 STO − 1 =	**6,500.00**	Total seventh payment.
CLx RCL 0 −	RCL 1	**25,000.00**	Remaining balance.
x≷y %	X RCL 2 % +	**1,250.00**	Eighth payment's interest.
RCL 0 +	RCL 0 STO − 1 =	**6,250.00**	Total eighth payment.
CLx RCL 0 −	RCL 1	**20,000.00**	Remaining balance after fourth year.

Add-On Interest Rate Converted to APR

An add-on interest rate determines what portion of the principal will be added on for repayment of a loan. This sum is then divided by the number of months in a loan to determine the monthly payment. For example, a 10% add-on rate for 36 months on $3000 means add one-tenth of $3000 for 3 years (300 x 3) - usually called the "finance charge" - for a total of $3900. The monthly payment is $3900/36.

This keystroke procedure converts an add-on interest rate to a annual percentage rate when the add-on rate and number of months are known.

RPN Mode:

1. Press [g] [END] and press [f] CLEAR [FIN] .
2. Key in the number of months in the loan and press [n] [ENTER] [RCL] [g] [12x] .
3. Key in the add-on rate and press [X].
4. Key in the amount of the loan and press [PV] * [x≷y] [%] [+].
5. Press [x≷y] [÷] [CHS] [PMT].
6. Press [i] 12 [X] to obtain the APR.

ALG Mode:

1. Press [g] [END] and press [f] CLEAR [FIN] .
2. Key in the number of months in the loan and press [n] [=] [RCL] [g] [12x] [X] .
3. Key in the add-on rate and press [=] .
4. Key in the amount of the loan and press [PV] * [+] [x≷y] [%] [÷] .
5. Press [x≷y] [=] [CHS] [PMT] .
6. Press [i] [X] 12 [=] to obtain the APR.

Example 1: Calculate the APR and monthly payment of a 12% $1000 add-on loan which has a life of 18 months.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
[g] [END] [f] CLEAR [FIN] 18 [n] [ENTER] [RCL] [g] [12x] 12 [X] 1000 [PV] [x≷y] [%] [+]	[g] [END] [f] CLEAR [FIN] 18 [n] [=] [RCL] [g] [12x] [X] 12 [=] 1000 [PV] [+] [x≷y] [%] [÷]	**1,180.00**	Amount of loan.
[x≷y] [÷] [CHS] [PMT]	[x≷y] [=] [CHS] [PMT]	**-65.56**	Monthly payment.
[i] 12 [X]	[i] [X] 12 [=]	**21.64**	Annual Percentage Rate.

APR Converted to Add-On Interest Rate

Given the number of months and annual percentage rate, this procedure calculates the corresponding add-on interest rate.

1. Press [g] [END] and press [f] CLEAR [FIN] .

* Positive for cash received; negative for cash paid out.

2. Enter the following information:
   a. Key in number of months of loan and press [n] .
   b. Key in APR and press [g] [12÷] .
   c. Key in 100 and press [PV] [PMT] .

3. **RPN:** Press [RCL] [PV] [RCL] [n] [÷] [+] [CHS] 12 [X] to obtain the add-on rate.

3. **ALG:** Press [RCL] [PV] [÷] [RCL] [n] [+] [x≷y] [X] 12 [=] [CHS] to obtain the add-on rate.

Example 1: What is the equivalent add-on rate for an 18 month loan with an APR of 14%?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
[g] [END] [f] CLEAR [FIN] 18 [n] 14 [g] [12÷] 100 [PV] [PMT] [RCL] [PV] [RCL] [n] [÷] [+] [CHS] 12 [X]	[g] [END] [f] CLEAR [FIN] 18 [n] 14 [g] [12÷] 100 [PV] [PMT] [RCL] [PV] [÷] [RCL] [n] [+] [x≷y] [X] 12 [=] [CHS]	**7.63**	Add-On Interest Rate.

Add-On Rate Loan with Credit Life

This HP 12C Platinum program calculates the monthly payment amount, credit life amount (an optional insurance which cancels any remaining indebtedness at the death of the borrower), total finance charge, and annual percentage rate (APR) for an add-on interest rate (AIR) loan. The monthly payment is rounded (in normal manner) to the nearest cent. If other rounding techniques are used, slightly different results may occur.

12c platinum / 12C RPN KEYSTROKES	DISPLAY	12c platinum ALG KEYSTROKES	DISPLAY
[f] [P/R]		[f] [P/R]	
[f] CLEAR [PRGM]	000,	[f] CLEAR [PRGM]	000,
[g] [END]	001, 43 8	[g] [END]	001, 43 8
1	002, 1	[RCL] 0	002, 45 0
[RCL] 0	003, 45 0	[n]	003, 11
1	004, 1	[RCL] [g] [12X]	004,45,43 11
2	005, 2	[%]	005, 25
0	006, 0	[STO] 4	006, 44 4
0	007, 0	[X]	007, 20
[÷]	008, 10	[RCL] 2	008, 45 2
[STO] 4	009, 44 4	[=]	009, 36
[RCL] 2	010, 45 2	[RCL] 4	010, 45 4

12c platinum / 12C RPN KEYSTROKES	DISPLAY
X	011, 20
−	012, 30
g LSTx	013, 43 40
RCL 1	014, 45 1
X	015, 20
RCL 4	016, 45 4
X	017, 20
−	018, 30
RCL 4	019, 45 4
RCL 1	020, 45 1
X	021, 20
1	022, 1
+	023, 40
x≷y	024, 34
÷	025, 10
RCL 3	026, 45 3
X	027, 20
RCL 0	028, 45 0
÷	029, 10
f RND	030, 42 14
CHS	031, 16
PMT	032, 14
R/S	033, 31
RCL PMT	034, 45 14
RCL 0	035, 45 0
X	036, 20
CHS	037, 16
PV	038, 13
RCL PV	039, 45 13
RCL 2	040, 45 2
%	041, 25
RCL 0	042, 45 0
X	043, 20
1	044, 1
2	045, 2
÷	046, 10
STO 5	047, 44 5
EEX	048, 26
2	049, 2
X	050, 20
g FRAC	051, 43 24
g x=0	052, 43 35
g GTO 061	053,43,33,061
RCL 5	054, 45 5

12c platinum ALG KEYSTROKES	DISPLAY
X	011, 20
RCL 1	012, 45 1
+	013, 40
1	014, 1
=	015, 36
1/x	016, 22
−	017, 30
x≷y	018, 34
X	019, 20
RCL 0	020, 45 0
÷	021, 10
RCL 3	022, 45 3
=	023, 36
1/x	024, 22
f RND	025, 42 14
CHS	026, 16
PMT	027, 14
R/S	028, 31
CHS	029, 16
X	030, 20
RCL 0	031, 45 0
X	032, 20
RCL 2	033, 45 2
X	034, 20
RCL 4	035, 45 4
=	036, 36
STO 5	037, 44 5
X	038, 20
EEX	039, 26
2	040, 2
=	041, 36
g FRAC	042, 43 24
g x=0	043, 43 35
g GTO 052	044,43,33,052
1	045, 1
%	046, 25
+	047, 40
RCL 5	048, 45 5
=	049, 36
f RND	050, 42 14
STO 5	051, 44 5
RCL 5	052, 45 5
R/S	053, 31
+	054, 40

12c platinum / 12C RPN KEYSTROKES	DISPLAY
•	055, 48
0	056, 0
1	057, 1
+	058, 40
f RND	059, 42 14
STO 5	060, 44 5
RCL 5	061, 45 5
R/S	062, 31
RCL PV	063, 45 13
x≷y	064, 34
−	065, 30
RCL 3	066, 45 3
−	067, 30
CHS	068, 16
R/S	069, 31
RCL 5	070, 45 5
RCL 3	071, 45 3
+	072, 40
PV	073, 13
RCL 0	074, 45 0
n	075, 11
i	076, 12
RCL g 12÷	077,45,43 12
g GTO 000	078,43,33,000
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
RCL 3	055, 45 3
PV	056, 13
FV	057, 15
CHS	058, 16
R/S	059, 31
0	060, 0
FV	061, 15
i	062, 12
RCL g 12÷	063,45,43 12
g GTO 000	064,43,33,000
f P/R	

	REGISTERS		
n: *N*	i: *i*	PV: Used	PMT: *PMT*
FV: 0	R_0: *N*	R_1: *AIR*	R_2: *CL* (%)
R_3: Loan	R_4: *N*/1200	R_5: Used	R_6-R_9: Unused

Program Instructions:

1. Key in the program.
2. Press f CLEAR FIN.
3. Key in the number of monthly payments in the loan and press STO 0.
4. Key in the annual add-on interest rate as a percentage and press STO 1.
5. Key in the credit life as a percentage and press STO 2.
6. Key in the loan amount and press STO 3.
7. Press R/S to find the monthly payment amount.
8. Press R/S to obtain the amount of credit life.

9. Press [R/S] to calculate the total finance charge.
10. Press [R/S] to calculate the annual percentage rate.
11. For a new loan return to step 3.

Example 1: You wish to quote a loan on a $3100 balance, payable over 36 months at an add-on rate of 6.75%. Credit life (CL) is 1%. What are the monthly payment amount, credit life amount, total finance charge, and APR?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
[f] CLEAR [FIN]	[f] CLEAR [FIN]		
36 [STO] 0	36 [STO] 0	**36.00**	Months.
6.75 [STO] 1	6.75 [STO] 1	**6.75**	Add-on interest rate.
1 [STO] 2	1 [STO] 2	**1.00**	Credit life (%).
3100 [STO] 3	3100 [STO] 3	**3100.00**	Loan.
[R/S]	[R/S]	**-107.42**	Monthly payment.
[R/S]	[R/S]	**116.02**	Credit life.
[R/S]	[R/S]	**-651.10**	Total finance charge.
[R/S]	[R/S]	**12.39**	APR.

Interest Rebate - Rule of 78's

This procedure finds the unearned interest rebate, as well as the remaining principal balance due for a prepaid consumer loan using the Rule of 78's. The known values are the current installment number, the total number of installments for which the loan was written, and the total finance charge (amount of interest). The information is entered as follows:

RPN Mode:

1. Key in number of months in the loan and press [STO] 1.
2. Key in payment number when prepayment occurs and press [−] [STO] 2 1 [+].
3. Key in total finance charge and press [X] [RCL] 1 [ENTER] [X] [RCL] 1 [+] [÷] [RCL] 2 [X] to obtain the unearned interest (rebate).
4. Key in periodic payment amount and press [RCL] 2 [X] [x≷y] [−] to obtain the amount of principal outstanding.

ALG Mode:

1. Key in number of months in the loan and press [STO] 1 [−].
2. Key in payment number when prepayment occurs and press [=] [STO] 2 [+] 1 [X].
3. Key in total finance charge and press [=] [RCL] 1 [g] [x^2] [+] [RCL] 1 [÷] [x≷y] [x≷y] [X] [RCL] 2 [=] to obtain the unearned interest (rebate).
4. Key in periodic payment amount and press [X] [RCL] 2 [−] [x≷y] [=] to obtain the amount of principal outstanding.

Example 1: A 30 month $1000 loan having a finance charge of $180, is being repaid at $39.33 per month. What is the rebate and balance due after the 25th regular payment?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
30 STO 1 25 − STO 2 1 + 180 X RCL 1 ENTER X RCL 1 + ÷ RCL 2 X	30 STO 1 − 25 = STO 2 + 1 X 180 = RCL 1 g x^2 + RCL 1 ÷ x≷y x≷y X RCL 2 =	**5.81**	Rebate.
39.33 RCL 2 X x≷y −	39.33 X RCL 2 − x≷y =	**190.84**	Outstanding principal.

The following HP 12C Platinum program can be used to evaluate the previous example.

12c platinum / 12C RPN KEYSTROKES	DISPLAY	12c platinum ALG KEYSTROKES	DISPLAY
f P/R		f P/R	
f CLEAR PRGM	000,	f CLEAR PRGM	000,
STO 0	001, 44 0	STO 0	001, 44 0
R↓	002, 33	R↓	002, 33
STO 2	003, 44 2	x≷y	003, 34
R↓	004, 33	STO 1	004, 44 1
STO 1	005, 44 1	STO 2	005, 44 2
RCL 2	006, 45 2	x≷y	006, 34
−	007, 30	STO − 2	007,44 30 2
STO 2	008, 44 2	RCL 1	008, 45 1
1	009, 1	g x^2	009, 43 20
+	010, 40	+	010, 40
RCL 0	011, 45 0	RCL 1	011, 45 1
X	012, 20	=	012, 36
RCL 1	013, 45 1	RCL 2	013, 45 2
ENTER	014, 36	g x^2	014, 43 20
X	015, 20	+	015, 40
RCL 1	016, 45 1	RCL 2	016, 45 2
+	017, 40	÷	017, 10
÷	018, 10	x≷y	018, 34
RCL 2	019, 45 2	X	019, 20
X	020, 20	RCL 0	020, 45 0
R/S	021, 31	=	021, 36
RCL 2	022, 45 2	R/S	022, 31
X	023, 20	X	023, 20
x≷y	024, 34	RCL 2	024, 45 2
−	025, 30	−	025, 30

12c platinum / 12C RPN KEYSTROKES	DISPLAY
g GTO 000	026,43,33,000
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
x≷y	026, 34
=	027, 36
g GTO 000	028,43,33,000
f P/R	

	REGISTERS		
N: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: Fin. charge	R_1: # months	R_2: Payment #
R_3-R_6: Unused			

Program Instructions:

1. Key in the program.
2. Key in the number of months in the loan and press ENTER (=).
3. Key in the payment number when prepayment occurs and press ENTER (=).
4. Key in the total finance charge and press R/S to obtain the unearned interest (rebate).
5. Key in the periodic payment amount and press R/S to find the amount of principal outstanding.
6. For a new case return to step 2.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
30 ENTER 25 ENTER 180 R/S	30 = 25 = 180 R/S	**5.81**	Rebate.
39.33 R/S	39.33 R/S	**190.84**	Outstanding principal.

Graduated Payment Mortgages

The Graduated Payment Mortgage is designed to meet the needs of young home buyers who currently cannot afford high mortgage payments, but who have the potential of increasing earning in the years to come.

Under the Graduated Payment Mortgage plan, the payments increase by a fixed percentage at the end of each year for a specified number of years. Thereafter, the payment amount remains constant for remaining life of the mortgage.

The result is that the borrower pays a reduced payment (a payment which is less than a traditional mortgage payment) in the early years, and in the later years makes larger payments than he would with a traditional loan. Over the entire term of the mortgage, the borrower would pay more than he would with conventional financing.

Given the term of the mortgage (in years), the annual percentage rate, the loan amount, the percentage that the payments increase, and the number of years that the payments increase, the following HP 12C Platinum program determines the monthly payments and remaining balance for each year until the level payment is reached.

12c platinum / 12C RPN KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
g END	001, 43 8
STO 2	002, 44 2
x≷y	003, 34
1	004, 1
%	005, 25
1	006, 1
+	007, 40
STO 0	008, 44 0
RCL n	009, 45 11
RCL 2	010, 45 2
−	011, 30
g 12x	012, 43 11
RCL i	013, 45 12
g 12÷	014, 43 12
RCL PV	015, 45 13
STO 3	016, 44 3
1	017, 1
CHS	018, 16
PMT	019, 14
PV	020, 13
CHS	021, 16
FV	022, 15
1	023, 1
g 12x	024, 43 11
RCL PMT	025, 45 14
RCL 0	026, 45 0
÷	027, 10
PMT	028, 14
PV	029, 13
CHS	030, 16
FV	031, 15
1	032, 1
STO + 1	033,44 40 1
RCL 1	034, 45 1
RCL 2	035, 45 2
−	036, 30

12c platinum ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
g END	001, 43 8
STO 2	002, 44 2
x≷y	003, 34
%	004, 25
+	005, 40
1	006, 1
=	007, 36
1/x	008, 22
STO 0	009, 44 0
RCL n	010, 45 11
−	011, 30
RCL 2	012, 45 2
=	013, 36
g 12x	014, 43 11
RCL i	015, 45 12
g 12÷	016, 43 12
RCL PV	017, 45 13
STO 3	018, 44 3
STO 4	019, 44 4
1	020, 1
PMT	021, 14
PV	022, 13
CHS	023, 16
FV	024, 15
1	025, 1
g 12x	026, 43 11
RCL PMT	027, 45 14
x	028, 20
RCL 0	029, 45 0
PMT	030, 14
PV	031, 13
CHS	032, 16
FV	033, 15
1	034, 1
STO + 1	035,44 40 1
RCL 1	036, 45 1

12c platinum / 12C RPN KEYSTROKES	DISPLAY
g x=0	037, 43 35
g GTO 040	038,43,33,040
g GTO 025	039,43,33,025
RCL 3	040, 45 3
RCL PV	041, 45 13
÷	042, 10
STO 4	043, 44 4
RCL 3	044, 45 3
PV	045, 13
1	046, 1
STO 3	047, 44 3
RCL 3	048, 45 3
R/S	049, 31
RCL 4	050, 45 4
1	051, 1
RCL 0	052, 45 0
RCL 1	053, 45 1
y^x	054, 21
÷	055, 10
X	056, 20
CHS	057, 16
f RND	058, 42 14
PMT	059, 14
R/S	060, 31
FV	061, 15
FV	062, 15
f RND	063, 42 14
R/S	064, 31
CHS	065, 16
PV	066, 13
1	067, 1
STO + 3	068,44 40 3
STO − 1	069,44 30 1
RCL 1	070, 45 1
g x=0	071, 43 35
g GTO 074	072,43,33,074
g GTO 048	073,43,33,048
RCL 4	074, 45 4
CHS	075, 16
R/S	076, 31
g GTO 076	077,43,33,076
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
−	037, 30
RCL 2	038, 45 2
=	039, 36
g x=0	040, 43 35
g GTO 043	041,43,33,043
g GTO 027	042,43,33,027
RCL PV	043, 45 13
STO ÷ 4	044,44 10 4
RCL 3	045, 45 3
PV	046, 13
1	047, 1
STO 3	048, 44 3
RCL 3	049, 45 3
R/S	050, 31
RCL 0	051, 45 0
y^x	052, 21
RCL 1	053, 45 1
X	054, 20
RCL 4	055, 45 4
=	056, 36
f RND	057, 42 14
R/S	058, 31
PMT	059, 14
FV	060, 15
f RND	061, 42 14
R/S	062, 31
CHS	063, 16
PV	064, 13
1	065, 1
STO + 3	066,44 40 3
STO − 1	067,44 30 1
RCL 1	068, 45 1
g x=0	069, 43 35
g GTO 072	070,43,33,072
g GTO 049	071,43,33,049
RCL 4	072, 45 4
R/S	073, 31
g GTO 073	074,43,33,073
f P/R	

	REGISTERS		
n: Used	i: i/12	PV: Used	PMT: Used
FV: Used	R_0: Used	R_1: Used	R_2: Used
R_3: Used	R_4: Level Pmt.	R_5-R_9: Unused	

Program Instructions:

1. Key in the program.
2. Press [f] CLEAR [REG].
3. Key in the term of the loan and press [n].
4. Key in the annual interest rate and press [i].
5. Key in the total loan amount and press [PV].
6. Key in the rate of graduation (as a percent) and press [ENTER] ([=]).
7. Key in the number of years for which the loan graduates and press [R/S]. The following information will be displayed for each year until a level payment is reached.
   a. The current year.
      Then press [R/S] to continue.
   b. The monthly payment for the current year.
      Then press [R/S] to continue.
   c. The remaining balance to be paid on the loan at the end of the current year.
      Then press [R/S] to return to step a. unless the level payment is reached.
      If the level payment has been reached, the program will stop, displaying the monthly payment over the remaining term of the loan.
8. For a new case press [g] [GTO] 000 and return to step 2.

Example: A young couple recently purchased a new house with a Graduated Payment Mortgage. The loan is for $50,000 over a period of 30 years at an annual interest rate of 12.5%. The monthly payments will be graduating at an annual rate of 5% for the first 5 years and then will be level for the remaining 25 years. What are the monthly payment amount for the first 6 years?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
[f] CLEAR [REG]	[f] CLEAR [REG]	**0.00**	
30 [n]	30 [n]	**30.00**	Term
12.5 [i]	12.5 [i]	**12.50**	Annual interest rate
50000 [PV]	50000 [PV]	**50,000.00**	Loan amount
5 [ENTER]	5 [=]	**5.00**	Rate of graduation
5 [R/S]	5 [R/S]	**1.00**	Year 1
[R/S]	[R/S]	**-448.88**	1st year monthly payment.
[R/S]	[R/S]	**-50,914.67**	Remaining balance after 1st year.
[R/S]	[R/S]	**2.00**	Year 2
[R/S]	[R/S]	**-471.33**	2nd year monthly payment.
[R/S]	[R/S]	**-51,665.07**	Remaining balance after 2nd year.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
R/S R/S R/S	R/S R/S R/S	**3.00** **-494.89** **-52,215.34**	Year 3 3rd year monthly payment. Remaining balance after 3rd year.
R/S R/S R/S	R/S R/S R/S	**4.00** **-519.64** **-52,523.86**	Year 4 4th year monthly payment. Remaining balance after 4th year.
R/S R/S R/S	R/S R/S R/S	**5.00** **-545.62** **-52,542.97**	Year 5 5th year monthly payment. Remaining balance after 5th year.
R/S	R/S	**-572.90**	Monthly payment for remainder of term.

Variable Rate Mortgages

As its name suggests, a variable rate mortgage is a mortgage loan which provides for adjustment of its interest rate as market interest rates change. As a result, the current interest rate on a variable rate mortgage may differ from its origination rate (i.e., the rate when the loan was made). This is the difference between a variable rate mortgage and the standard fixed payment mortgage, where the interest rate and the monthly payment are constant throughout the term.

Under the agreement of the variable rate mortgage, the mortgage is examined periodically to determine any rate adjustments. The rate adjustment may be implemented in two ways:

1. Adjusting the monthly payment.
2. Modifying the term of the mortgage.

The period and limits to interest rate increases vary from state to state.

Each periodic adjustment may be calculated by using the HP 12C Platinum with the following keystroke procedure. The original terms of the mortgage are assumed to be known.

1. Press g END and press f CLEAR FIN.
2. Key in the remaining balance of the loan and press PV.

   The remaining balance is the difference between the loan amount and the total principal from the payments which have been made.
   To calculate the remaining balance, do the following:
   a. Key in the previous remaining balance. If this is the first mortgage adjustment, this value is the original amount of the loan. Press PV.
   b. Key in the annual interest rate before the adjustment (as a percentage) and press g 12÷.

c. Key in the number of years since the last adjustment. If this is the first mortgage adjustment, then key in the number of years since the origination of the mortgage. Press [g] [12x].

d. Key in the monthly payment over this period and press [CHS] [PMT].

e. Press [FV] to find the remaining balance, then press [f] CLEAR [FIN] [CHS] [PV].

3. Key in the adjusted annual interest rate (as a percentage) and press [g] [12÷].

To calculate the new monthly payment:

a. Key in the remaining life of the mortgage (years) and press [g] [12x].

b. **RPN:** Press [PMT] to find the new monthly payment.

b. **ALG:** Press [PMT] [PMT] to find the new monthly payment.

To calculate the revised remaining term of the mortgage:

a. Key in the present monthly payment and press [PMT].

b. **RPN:** Press [n] 12 [÷] to find the remaining term of the mortgage in years.

b. **ALG:** Press [n] [÷] 12 [=] to find the remaining term of the mortgage in years.

Example: A homeowner purchased his house 3 years ago with a $50,000 variable rate mortgage. With a 30 year term, his current monthly payment is $495.15. When the interest rate is adjusted from 11.5% to 11.75%, what will the monthly payment be? If the monthly payment remained unchanged, find the revised remaining term on the mortgage.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
[g] [END] [f] CLEAR [FIN] 50000 [PV]	[g] [END] [f] CLEAR [FIN] 50000 [PV]	**50,000.00**	Original amount of loan.
11.5 [g] [12÷]	11.5 [g] [12÷]	**0.96**	Original monthly interest rate.
3 [g] [12x]	3 [g] [12x]	**36.00**	Period.
495.15 [CHS] [PMT]	495.15 [CHS] [PMT]	**-495.15**	Previous monthly payment.
[FV] [f] CLEAR [FIN] [CHS] [PV]	[FV] [f] CLEAR [FIN] [CHS] [PV]	**-49,316.74** **49,316.74**	Remaining balance.
11.75 [g] [12÷]	11.75 [g] [12÷]	**0.98**	Adjusted monthly interest.
30 [ENTER] 3 [−]	30 [−] 3 [=]	**27.00**	Remaining life of mortgage.
[g] [12x] [PMT]	[g] [12x] [PMT] [PMT]	**324.00** **-504.35**	New monthly payment.
495.15 [CHS] [PMT]	495.15 [CHS] [PMT]	**-495.15**	Previous monthly payment.
[n] 12 [÷]	[n] [÷] 12 [=]	**31.67**	New remaining term (years).

Skipped Payments

Sometimes a loan (or lease) may be negotiated in which a specific set of monthly payments are going to be skipped each year. Seasonally is usually the reason for such an agreement. For example, because of heavy rainfall, a bulldozer cannot be operated in Oregon during December, January, and February, and the lessee wishes to make payments only when his machinery is being used. He will make nine payments per year, but the interest will continue to accumulate over the months in which a payment is not made.

To find the monthly payment amount necessary to amortize the loan in the specified amount of time, information is entered as follows:

1. Press [g] [END] and press [f] CLEAR [FIN].
2. Key in the number of the last payment period before payments close the first time and press [n].
3. Key in the annual interest rate as a percentage and press [g] [12÷] 1 [PMT] [FV].

RPN Mode:

4. Press [CHS] [PV] 12 [RCL] [n] [−] [n] 0 [PMT] [FV] [STO] 0 [RCL] [n].
5. Key in the number of payments which are skipped and press [−] [n] 1 [PMT] 0 [PV] [FV] [STO] [+] 0.
6. Press 0 [PMT] 12 [n] 100 [PV] [FV] [RCL] [PV] [+] [CHS] [f] CLEAR [FIN] [i]
7. Key in the total number of years in the loan and press [n].
8. Key in the loan amount and press [PV] [PMT] [RCL] 0 [÷] to obtain the monthly payment amount when the payment is made at the end of the month.
9. Press [CHS] [FV] 0 [PMT] 1 [n].
10. Key in the annual interest rate as a percent and press [g] [12÷] [PV] to find the monthly payment amount when the payment is made at the beginning of the month.

ALG Mode:

4. Press [CHS] [PV] 12 [−] [RCL] [n] [n] 0 [PMT] [FV] [STO] 0 [RCL] [n] [−].
5. Key in the number of payments which are skipped and press [n] 1 [PMT] 0 [PV] [FV] [STO] [+] 0.
6. Press 0 [PMT] 12 [n] 100 [PV] [FV] [+] [RCL] [PV] [=] [CHS] [f] CLEAR [FIN] [i]
7. Key in the total number of years in the loan and press [n].
8. Key in the loan amount and press [PV] [PMT] [÷] [RCL] 0 [=] to obtain the monthly payment amount when the payment is made at the end of the month.
9. Press [CHS] [FV] 0 [PMT] 1 [n].
10. Key in the annual interest rate as a percent and press [g] [12÷] [PV] [PV] to find the monthly payment amount when the payment is made at the beginning of the month.

Example: A bulldozer worth $100,000 is being purchased in September. The first payment is due one month later, and payments will continue over a period of 5 years. Due to the weather, the machinery will not be used during the winter months, and the purchaser does not wish to make payments during January, February, and March (months 4 thru 6). If the current interest rate is 14%, what is the monthly payment necessary to amortize the loan?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
g END f CLEAR FIN 3 n	g END f CLEAR FIN 3 n	**3.00**	Number of payments made before a group of payments is skipped.
14 g 12÷	14 g 12÷	**1.17**	Monthly interest rate.
1 PMT FV CHS PV 12 RCL n − n 0 PMT FV STO 0	1 PMT FV CHS PV 12 − RCL n n 0 PMT FV STO 0	**-3.37**	FV of 3 monthly unit PMTs.
RCL n 3 − n 1 PMT 0 PV FV STO + 0	RCL n − 3 n 1 PMT 0 PV FV STO + 0	**-6.18**	FV of 6 monthly unit PMTs.
0 PMT 12 n 100 PV FV RCL PV + CHS f CLEAR FIN i	0 PMT 12 n 100 PV FV + RCL PV = CHS f CLEAR FIN i	**14.93**	Effective annual interest rate
5 n 100000 PV PMT RCL 0 ÷	5 n 100000 PV PMT ÷ RCL 0 =	**3,119.89**	Monthly payment in arrears.

Savings

Initial Deposit with Periodic Deposits

Given an initial deposit into a savings account, and a series of periodic deposits coincident with the compounding period, the future value (or accumulated amount) may be calculated as follows:

1. Press [g] [END] and press [f] CLEAR [FIN].
2. Key in the initial investment and press [CHS] [PV].
3. Key in the number of additional periodic deposits and press [n].
4. Key in the periodic interest rate and press [i].
5. Key in the periodic deposit and press [CHS] [PMT].
6. Press [FV] to determine the value of the account at the end of the time period.

Example: You have just opened a savings account with a $200 deposit. If you deposit $50 a month, and the account earns 5 ¼ % compounded monthly, how much will you have in 3 years?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
[g] [END] [f] CLEAR [FIN] 200 [CHS] [PV] 3 [g] [12x] 5.25 [g] [12÷] 50 [CHS] [PMT] [FV]	[g] [END] [f] CLEAR [FIN] 200 [CHS] [PV] 3 [g] [12x] 5.25 [g] [12÷] 50 [CHS] [PMT] [FV]	**2,178.94**	Value of the account.

Note: If the periodic deposits do not coincide with the compounding periods, the account must be evaluated in another manner. First, find the future value of the initial deposits and store it. Then use the procedure for compounding periods different from payment periods to calculate the future value of the periodic deposits. Recall the future value of the initial deposit and add to obtain the value of the account.

Number of Periods to Deplete a Savings Account or to Reach a Specified Balance

Given the current value of a savings account, the periodic interest rate, the amount of the periodic withdrawal, and a specified balance, this procedure determines the number of periods to reach that balance (the balance is zero if the account is depleted).

1. Press [g] [END] and press [f] CLEAR [FIN].
2. Key in the value of the savings account and press [CHS] [PV].
3. Key in the periodic interest rate and press [i].
4. Key in the amount of the periodic withdrawal and press [PMT].
5. Key in the amount remaining in the account and press [FV].
   This step may be omitted if the account is depleted (FV=0).
6. Press [n] to determine the number of periods to reach the desired balance.

Example: Your savings account presently contains $18,000 and earns 5 ½% compounded monthly. You wish to withdraw $300 a month until the account is depleted. How long will this take? If you wish to reduce the account to $5000, how many withdrawals can you make?

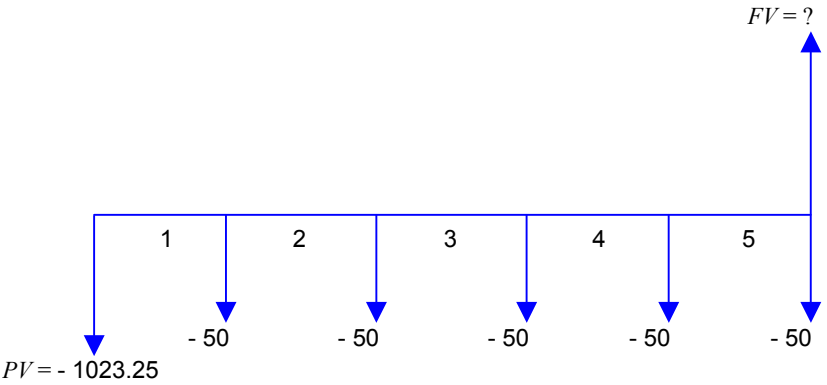
12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
[g] [END] [f] CLEAR [FIN] 18000 [CHS] [PV] 5.5 [g] [12÷] 300 [PMT] [n]	[g] [END] [f] CLEAR [FIN] 18000 [CHS] [PV] 5.5 [g] [12÷] 300 [PMT] [n]	**71.00**	Months to deplete account.
5000 [FV] [n]	5000 [FV] [n]	**53.00**	Months to reduce the account to $5000.

Periodic Deposits and Withdrawals

This section is presented as a guideline for evaluating a savings plan when deposits and withdrawals occur at irregular intervals. One problem is given, and a step by step method for setting up and solving the problem is presented:

Example: You are presently depositing $50 and the end of each month into a local savings and loan, earning 5 ½% compounded monthly. Your current balance is $1023.25. How much will you have accumulated in 5 months?

The cash flow diagram looks like this:



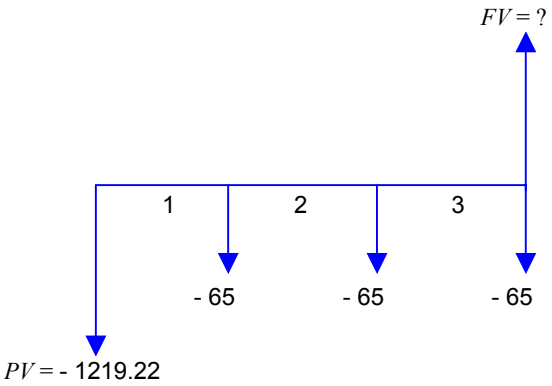
12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
g END f CLEAR FIN 50 CHS PMT 5.5 g 12÷ 1023.25 CHS PV 5 n FV	g END f CLEAR FIN 50 CHS PMT 5.5 g 12÷ 1023.25 CHS PV 5 n FV	**1,299.22**	Amount in account.

Now suppose that at the beginning of the 6th month you withdrew $80.

What is the new balance?

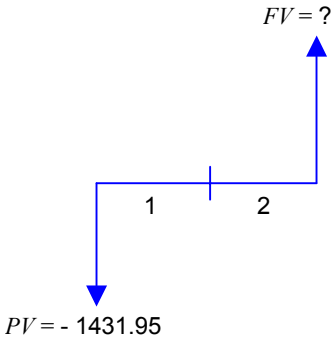
12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
80 −	− 80 =	**1,219.22**	New balance.

You increase your monthly deposit to $65. How much will you have in 3 months?
The cash flow diagram looks like this:



12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
CHS PV 65 CHS PMT 3 n FV	CHS PV 65 CHS PMT 3 n FV	**1,431.95**	Account balance.

Suppose that for 2 months you decide not to make a periodic deposit. What is the balance in the account?



12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
CHS PV 2 n 0 PMT FV	CHS PV 2 n 0 PMT FV	**1,445.11**	Account balance.

This type of procedure may be continued for any length of time, and may be modified to meet the user's particular needs.

Savings Account Compounded Daily

This HP 12C Platinum program determines the value of a savings account when interest is compounded daily, based on a 365 day year. The user is able to calculate the total amount remaining in the account after a series of transactions on specified dates.

12c platinum / 12C RPN KEYSTROKES	DISPLAY	12c platinum ALG KEYSTROKES	DISPLAY
f P/R		f P/R	
f CLEAR PRGM	000,	f CLEAR PRGM	000,
CHS	001, 16	CHS	001, 16
PV	002, 13	PV	002, 13
R↓	003, 33	R↓	003, 33
3	004, 3	÷	004, 10
6	005, 6	3	005, 3
5	006, 5	6	006, 6
÷	007, 10	5	007, 5
i	008, 12	i	008, 12
R↓	009, 33	R↓	009, 33
STO 0	010, 44 0	STO 0	010, 44 0
RCL PV	011, 45 13	RCL PV	011, 45 13
CHS	012, 16	CHS	012, 16
R/S	013, 31	R/S	013, 31
STO 2	014, 44 2	STO 2	014, 44 2
R↓	015, 33	R↓	015, 33
STO 1	016, 44 1	STO 1	016, 44 1
RCL 0	017, 45 0	RCL 0	017, 45 0
RCL 1	018, 45 1	RCL 1	018, 45 1
g ΔDYS	019, 43 26	g ΔDYS	019, 43 26
n	020, 11	n	020, 11
FV	021, 15	FV	021, 15
f RND	022, 42 14	f RND	022, 42 14
FV	023, 15	FV	023, 15
ENTER	024, 36	+	024, 40
RCL PV	025, 45 13	RCL PV	025, 45 13
+	026, 40	=	026, 36
STO + 3	027,44 40 3	STO + 3	027,44 40 3
RCL FV	028, 45 15	RCL FV	028, 45 15
RCL 2	029, 45 2	CHS	029, 16
+	030, 40	−	030, 30
CHS	031, 16	RCL 2	031, 45 2

12c platinum / 12C RPN KEYSTROKES	DISPLAY
PV	032, 13
RCL 1	033, 45 1
STO 0	034, 44 0
RCL PV	035, 45 13
CHS	036, 16
g GTO 013	037,43,33,013
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
PV	032, 13
RCL 1	033, 45 1
STO 0	034, 44 0
RCL PV	035, 45 13
CHS	036, 16
g GTO 013	037,43,33,013
f P/R	

	REGISTERS		
N: Δdays	i: i/365	PV: Used	PMT: 0
FV: Used	R_0: Initial date	R_1: Next date	R_2: $ amount
R_3: Interest	R_4-R_4: Unused		

Program Instructions:

1. Key in the program.
2. Press f CLEAR REG and press g M.DY.
3. Key in the date (MM.DDYYYY) of the first transaction and press ENTER (=).
4. Key in the annual nominal interest rate as a percentage and press ENTER (=).
5. Key in the amount of the initial deposit and press R/S.
6. Key in the date of the next transaction and press ENTER (=).
7. Key in the amount of the transaction (positive for money deposited, negative for cash withdrawn) and press R/S to determine the amount in the account.
8. Repeat steps 6 and 7 for subsequent transactions.
9. To see the total interest to date, press RCL 3.
10. For a new case press f GTO 000 and go to step 2.

Example: Compute the amount remaining in this 5.25% account after the following transactions:

1. January 19, 2003 deposit $125.00
2. February 24, 2003 deposit $60.00
3. March 16, 2003 deposit $70.00
4. April 6, 2003 withdraw $50.00
5. June 1, 2003 deposit $175.00

6 July 6, 2003 withdraw $100.00

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
f CLEAR REG g M.DY 1.192003 ENTER 5.25 ENTER 125 R/S	f CLEAR REG g M.DY 1.192003 = 5.25 = 125 R/S	**125.00**	Initial Deposit.
2.242003 ENTER 60 R/S	2.242003 = 60 R/S	**185.65**	Balance in account, February 24, 2003.
3.162003 ENTER 70 R/S	3.162003 = 70 R/S	**256.18**	Balance in account, March 16, 2003.
4.062003 ENTER 50 CHS R/S	4.062003 = 50 CHS R/S	**206.95**	Balance in account, April 6, 2003.
6.012003 ENTER 175 R/S	6.012003 = 175 R/S	**383.62**	Balance in account, June 1, 2003.
7.062003 ENTER 100 CHS R/S	7.062003 = 100 CHS R/S	**285.56**	Balance in account, July 6, 2003.
RCL 3	RCL 3	**5.56**	Total interest.

Compounding Periods Different From Payment Periods

In financial calculations involving a series of payments equally spaced in time with periodic compounding, both periods of time are normally equal and coincident. This assumption is preprogrammed into the HP 12C Platinum.

In savings plans however, money may become available for deposit or investment at a frequency different from the compounding frequencies offered. The HP 12C Platinum can easily be used in these calculations. However, because of the assumptions mentioned the periodic interest rate must be adjusted to correspond to an equivalent rate for the payment period.

Payments deposited for a partial compounding period will accrue simple interest for the remainder of the compounding period. This is often the case, but may not be true for all institutions.

These procedures present solutions for future value, payment amount, and number of payments. In addition, it should be noted that only annuity due (payments at the beginning of payment period) calculations are shown since this is the most common in savings plan calculations.

To calculate the equivalent payment period interest rate, information is entered as follows:

1. Press [g] [BEG] and press [f] CLEAR [FIN].

RPN Mode:

2. Key in the annual interest rate (as a percent) and press [ENTER].
3. Key in the number of compounding periods per year and press [n] [÷] [i].

ALG Mode:

2. Key in the number of compounding periods per year and press [n].
3. Key in the annual interest rate (as a percent) and press [÷] [RCL] [n] [i].

4. Key in 1 and press [PV] [FV].
5. Key in the number of payments (deposits) per year and press [n] [i] [f] CLEAR [FIN] [i].

The interest rate which corresponds to the payment period is now in register "i" and you are ready to proceed.

Example 1: Solving for future value.

Starting today you make monthly deposits of $25 into an account paying 5% compounded daily (365-day basis). At the end of 7 years, how much will you receive from the account?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
[g] [BEG] [f] CLEAR [FIN] 5 [ENTER] 365 [n] [÷] [i] 1 [PV] [FV] 12 [n] [i] [f] CLEAR [FIN] [i]	[g] [BEG] [f] CLEAR [FIN] 365 [n] 5 [÷] [RCL] [n] [i] 1 [PV] [FV] 12 [n] [i] [f] CLEAR [FIN] [i]	**0.42**	Equivalent periodic interest rate.
7 [g] [12x] 25 [CHS] [PMT] [FV]	7 [g] [12x] 25 [CHS] [PMT] [FV]	**2,519.61**	Future value.

Example 2: Solving for payment amount.

For 8 years you wish to make weekly deposits in a savings account paying 5.5% compounded quarterly. What amount must you deposit each week to accumulate $6000.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
g BEG f CLEAR FIN 5.5 ENTER 4 n ÷ i 1 PV FV 52 n i f CLEAR FIN i	g BEG f CLEAR FIN 4 n 5.5 ÷ RCL n i 1 PV FV 52 n i f CLEAR FIN i	**0.11**	Equivalent periodic interest rate.
8 ENTER 52 X n 6000 FV PMT	8 X 52 n 6000 FV PMT	**-11.49**	Periodic payment.

Example 3: Solving for number of payment periods.

You can make weekly deposits of $10 in to an account paying 5.25% compounded daily (365-day basis). How long will it take you to accumulate $1000?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
g BEG f CLEAR FIN 5.25 ENTER 365 n ÷ i 1 PV FV 52 n i f CLEAR FIN i	g BEG f CLEAR FIN 365 n 5.25 ÷ RCL n i 1 PV FV 52 n i f CLEAR FIN i	**0.10**	Equivalent periodic interest rate.
10 CHS PMT 1000 FV n	10 CHS PMT 1000 FV n	**96.00**	Weeks.

Investment Analysis

Lease vs. Purchase

An investment decision frequently encountered is the decision to lease or purchase capital equipment or buildings. Although a thorough evaluation of a complex acquisition usually requires the services of a qualified accountant, it is possible to simplify a number of the assumptions to produce a first approximation.
The following HP 12C Platinum program assumes that the purchase is financed with a loan and that the loan is made for the term of the lease. The tax advantages of interest paid, depreciation, and the investment credit which accrues from ownership are compared to the tax advantage of treating the lease payment as an expense. The resulting cash flows are discounted to the present at the firm's after-tax cost of capital.

12c platinum / 12C RPN KEYSTROKES	DISPLAY		
f P/R			
f CLEAR PRGM	000,		
−	001,		30
1	002,		1
STO + 0	003,44	40	0
RCL 3	004,	45	3
−	005,		30
X	006,		20
STO 8	007,	44	8
1	008,		1
f AMORT	009,	42	11
STO 1	010,	44	1
RCL PV	011,	45	13
STO 9	012,	44	9
RCL PMT	013,	45	14
STO • 0	014,44	48	0
RCL n	015,	45	11
STO • 1	016,44	48	1
RCL i	017,	45	12
STO • 2	018,44	48	2
RCL 5	019,	45	5
PV	020,		13
RCL 6	021,	45	6
n	022,		11

12c platinum ALG KEYSTROKES	DISPLAY		
f P/R			
f CLEAR PRGM	000,		
−	001,		30
x≷y	002,		34
=	003,		36
STO 8	004,	44	8
1	005,		1
STO + 0	006,44	40	0
f AMORT	007,	42	11
STO 1	008,	44	1
RCL PV	009,	45	13
STO 9	010,	44	9
RCL i	011,	45	12
STO • 2	012,44	48	2
RCL 5	013,	45	5
PV	014,		13
RCL 6	015,	45	6
n	016,		11
RCL 7	017,	45	7
i	018,		12
RCL 0	019,	45	0
f SOYD	020,	42	24
STO + 1	021,44	40	1
RCL 9	022,	45	9

12c platinum / 12C RPN KEYSTROKES	DISPLAY
RCL 7	023, 45 7
i	024, 12
RCL 0	025, 45 0
f SOYD	026, 42 24
STO + 1	027,44 40 1
RCL 9	028, 45 9
PV	029, 13
RCL • 0	030,45 48 0
PMT	031, 14
RCL • 1	032,45 48 1
n	033, 11
RCL • 2	034,45 48 2
i	035, 12
RCL 1	036, 45 1
RCL 3	037, 45 3
X	038, 20
RCL PMT	039, 45 14
−	040, 30
RCL 8	041, 45 8
−	042, 30
RCL 4	043, 45 4
RCL 0	044, 45 0
y^x	045, 21
÷	046, 10
STO + 2	047,44 40 2
g GTO 000	048,43,33,000
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
PV	023, 13
RCL • 2	024,45 48 2
i	025, 12
RCL 1	026, 45 1
−	027, 30
RCL 8	028, 45 8
X	029, 20
RCL 3	030, 45 3
+	031, 40
RCL 8	032, 45 8
−	033, 30
RCL PMT	034, 45 14
=	035, 36
RCL 4	036, 45 4
y^x	037, 21
RCL 0	038, 45 0
÷	039, 10
x≷y	040, 34
x≷y	041, 34
=	042, 36
STO + 2	043,44 40 2
g GTO 000	044,43,33,000
f P/R	

	REGISTERS		
n: Used	i: Used	PV: Used	PMT: Used
FV: 0	R_0: Used	R_1: Used	R_2: Purch. Adv.
R_3: Tax	R_4: Discount	R_5: Dep. Value	R_6: Dep. life
R_7: Factor (DB)	R_8: Used	R_9: Used	$R_{.0}$: Used
$R_{.1}$: Used	$R_{.2}$: Used	$R_{.3}$: Unused	

Program Instructions:

1. Key in the program.

RPN: - Select the depreciation function and key in at line 26.
ALG: - Select the depreciation function and key in at line 20.

2. Press g END and press f CLEAR REG.
3. Input the following information for the purchase of the loan:
   - Key in the number of years for amortization and press n.

- Key in the annual interest rate and press [i].
- Key in the loan amount (purchase price) and press [CHS] [PV].
- Press [PMT] to find the annual payment.

4. Key in the marginal effective tax rate* and press [STO] 3.
5. **RPN:** Key in the discount rate or cost of capital* and press [ENTER] 1 [+] [STO] 4.
5. **ALG:** Key in the discount rate or cost of capital* and press [+] 1 [=] [STO] 4.
6. Key in the depreciable value and press [STO] 5.
7. Key in the depreciable life and press [STO] 6.
8. For declining balance depreciation, key in the depreciation factor (as a percentage) and press [STO] 7.
9. **RPN:** Key in the total first lease payment (including any advance payments) and press [ENTER] 1 [RCL] 3 [−] [X] [STO] 2.
9. **ALG:** Key in the total first lease payment (including any advance payments) and press [=] 1 [−] [RCL] 3 [X] [x≷y] [=] [STO] 2.
10. Key in the first year's maintenance expense that would be anticipated if the asset was owned and press [ENTER] ([=]) . If the lease contract does not include maintenance, then it is not a factor in the lease vs. purchase decision and 0 expense should be used.
11. Key in the next lease payment and press [R/S]. During any year in which a lease payment does not occur (e.g. the last several payments of an advance payment contract) use 0 for the payment.
12. Repeat steps 10 and 11 for all maintenance expenses and lease payments over the term of the analysis.
    Optional - If the investment tax credit is taken, key in the amount of the credit after finishing steps 10 and 11 for the year in which the credit is taken and press [g] [GTO] 043 (ALG:036) [R/S] . Continue steps 10 and 11 for the remainder of the term.
13. **RPN:** After all the lease payments and expenses have been entered (steps 10 and 11), key in the lease buy back option and press [ENTER] 1 [RCL] 3 [−] [X] [g] [GTO] 043 [R/S]. If no buy back option exists, use the estimated salvage value of the purchased equipment at the end of the term.
13. **ALG:** After all the lease payments and expenses have been entered (steps 10 and 11), key in the lease buy back option and press [=] 1 [−] [RCL] 3 [X] [x≷y] [=] [g] [GTO] 036 [R/S]. If no buy back option exists, use the estimated salvage value of the purchased equipment at the end of the term.
14. To find the net advantage of owning press [RCL] 2. A negative value represents a net lease advantage.

Example: Home Style Bagel Company is evaluating the acquisition of a mixer which can be leased for $1700 a year with the first and last payments in advance and a $750 buy back option at the end of 10 years (maintenance is included).
The same equipment could be purchased for $10,000 with a 12% loan amortized over 10 years. Ownership maintenance is estimated to be 2% of the purchase price per year for the first for years. A major overhaul is predicted for the 5th year at a cost of $1500. Subsequent yearly maintenance of 3% is estimated for the remainder of the 10 year term. The company would use sum of the years digits depreciation on a 10 year life with $1500

* Key in as a decimal (e.g., 5% as .05).

salvage value. An accountant informs management to take the 10% capital investment tax credit at the end of the second year and to figure the cash flows at a 48% tax rate. The after tax cost of capital (discounting rate) is 5 percent.
Because lease payments are made in advance and standard loan payments are made in arrears the following cash flow schedule is appropriate for a lease with the last payment in advance.

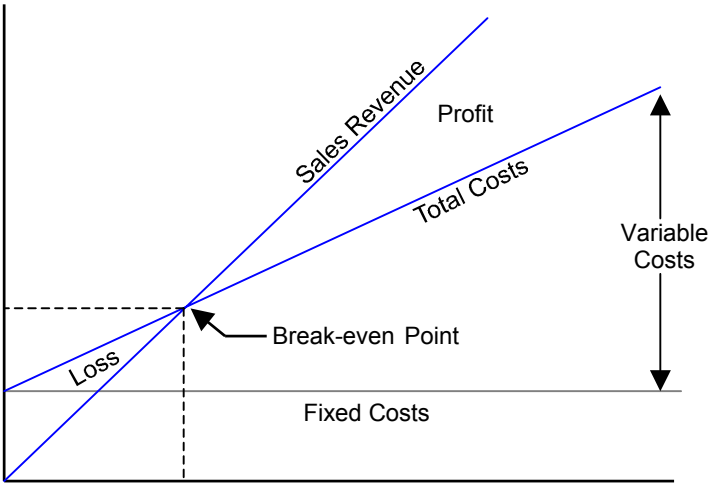
Year	Maintenance	Lease Payment	Tax Credit	Buy Back
0		1700+1700		
1	200	1700		
2	200	1700	1000	
3	200	1700		
4	200	1700		
5	1500	1700		
6	300	1700		
7	300	1700		
8	300	1700		
9	300	0		
10	300	0		750

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
g END f CLEAR REG 10 n 12 i 10000 CHS PV	g END f CLEAR REG 10 n 12 i 10000 CHS PV	**0.00** **-10,000.00**	Always use negative loan amount.
PMT	PMT	**1,769.84**	Purchase payment.
.48 STO 3	.48 STO 3	**0.48**	Marginal tax rate.
.05 ENTER 1 + STO 4	.05 + 1 = STO 4	**1.05**	Discounting factor.
10000 ENTER 1500 − STO 5	10000 − 1500 = STO 5	**8,500.00**	Depreciable value.
10 STO 6	10 STO 6	**10.00**	Depreciable life.
1700 ENTER +	1700 + =	**3,400.00**	1st lease payment.
1 RCL 3 − X STO 2	1 − RCL 3 X x≷y = STO 2	**1,768.00**	After-tax expense.
200 ENTER 1700 R/S	200 = 1700 R/S	**312.36**	Present value of 1st year's net purchase.
200 ENTER 1700 R/S	200 = 1700 R/S	**200.43**	2nd year's advantage.
1000 g GTO 043	1000 g GTO 036	**1,000.00**	Tax credit.
R/S	R/S	**907.03**	Present value of tax credit.
200 ENTER 1700 R/S	200 = 1700 R/S	**95.05**	3rd year.
200 ENTER 1700 R/S	200 = 1700 R/S	**-4.38**	4th year.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
1500 ENTER 1700 R/S	1500 = 1700 R/S	**-628.09**	5th year.
300 ENTER 1700 R/S	300 = 1700 R/S	**-226.44**	6th year.
300 ENTER 1700 R/S	300 = 1700 R/S	**-309.48**	7th year.
300 ENTER 1700 R/S	300 = 1700 R/S	**-388.81**	8th year.
300 ENTER 0 R/S	300 = 0 R/S	**-1,034.72**	9th year.
300 ENTER 0 R/S	300 = 0 R/S	**-1,080.88**	10th year.
750 ENTER	750 =	**750.00**	Buy back.
1 RCL 3 − X	1 − RCL 3 X x≷y =	**390.00**	After tax buy back expense.
g GTO 043 R/S	g GTO 036 R/S	**239.43**	Present value.
RCL 2	RCL 2	**-150.49**	Net lease advantage.

Break-Even Analysis

Break-even analysis is basically a technique for analyzing the relationships among fixed costs, variable costs, and income. Until the break-even point is reached at the intersection of the total sales revenue and total cost lines, the producer operates at a loss. After the break-even point each unit produced and sold makes a profit. Break-even analysis may be represented as follows.



The variables are: fixed costs (F), Sales price per unit (P), variable cost per unit (V), number of units sold (U), and gross profit (GP). One can readily evaluate GP, U or P given the four other variables. To calculate the break-even volume, simply let the gross profit equal zero and calculate the number of units sold (U).

To calculate the break-even volume:

RPN Mode:

1. Key in the fixed costs and press [ENTER].
2. Key in the unit price and press [ENTER].
3. Key in the variable cost per unit and press [−].
4. Press [÷] to calculate the break-even volume.

ALG Mode:

1. Key in the fixed costs and press [=].
2. Key in the unit price and press [−].
3. Key in the variable cost per unit and press [÷].
4. Press [x≷y] [x≷y] [=] to calculate the break-even volume.

To calculate the gross profit at a given volume:

RPN Mode:

1. Key in the unit price and press [ENTER].
2. Key in the variable cost per unit and press [−].
3. Key in the number of units sold and press [X].
4. Key in the fixed cost and press [−] to calculate the gross profit.

ALG Mode:

1. Key in the unit price and press [−].
2. Key in the variable cost per unit and press [X].
3. Key in the number of units sold and press [−].
4. Key in the fixed cost and press [=] to calculate the gross profit.

To calculate the sales volume needed to achieve a specified gross profit:

RPN Mode:

1. Key in the desired gross profit and press [ENTER].
2. Key in the fixed cost and press [+].
3. Key in sales price per unit and press [ENTER].
4. Key in the variable cost per unit and press [−].
5. Press [÷] to calculate the sales volume.

ALG Mode:

1. Key in the desired gross profit and press [+].
2. Key in the fixed cost and press [=].
3. Key in sales price per unit and press [−].
4. Key in the variable cost per unit and press [÷].
5. Press [x≷y] [x≷y] [=] to calculate the sales volume.

To calculate the required sales price to achieve a given gross profit at a specified sales volume:

RPN Mode:

1. Key in the fixed costs and press [ENTER].
2. Key in the gross desired and press [+].
3. Key in the specified sales volume in units and press [÷].
4. Key in the variable cost per unit and press [+] to calculate the required sales price per unit.

ALG Mode:

1. Key in the fixed costs and press [+].
2. Key in the gross desired and press [÷].
3. Key in the specified sales volume in units and press [+].
4. Key in the variable cost per unit and press [=] to calculate the required sales price per unit.

Example 1: The E.Z. Sells company markets textbooks on salesmanship. The fixed cost involved in setting up to print the books are $12,000. The variable cost per copy, including printing and marketing the books are $6.75 per copy. The sales price per copy is $13.00. How many copies must be sold to break even?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
12000 [ENTER]	12000 [=]	**12,000.00**	Fixed cost.
13 [ENTER]	13 [−]	**13.00**	Sales price.
6.75 [−] [÷]	6.75 [÷] [x≷y] [x≷y] [=]	**1,920.00**	Break-even volume.

Find the gross profit if 2500 units are sold.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
13 ENTER	13 −	**13.00**	Sales price.
6.75 −	6.75 ×	**6.25**	Profit per unit.
2500 × 12000 −	2500 − 12000 =	**15,625.00** **3,625.00**	Gross profit.

If a gross profit of $4500 is desired at a sales volume of 2500 units, what should the sales price be?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
12000 ENTER	12000 +	**12,000.00**	Fixed cost.
4500 + 2500 ÷ 6.75 +	4500 ÷ 2500 + 6.75 =	**16,500.00** **6.60** **13.35**	Sales price per unit to achieve desired gross profit.

For repeated calculation the following HP 12C Platinum program can be used.

12c platinum / 12C RPN KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
RCL 3	001, 45 3
RCL 2	002, 45 2
−	003, 30
g GTO 000	004,43,33,000
RCL 4	005, 45 4
×	006, 20
RCL 1	007, 45 1
−	008, 30
g GTO 000	009,43,33,000
RCL 5	010, 45 5
RCL 1	011, 45 1
+	012, 40
x≷y	013, 34
÷	014, 10
g GTO 000	015,43,33,000
RCL 1	016, 45 1
RCL 5	017, 45 5
+	018, 40

12c platinum ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
RCL 3	001, 45 3
−	002, 30
RCL 2	003, 45 2
=	004, 36
g GTO 000	005,43,33,000
×	006, 20
RCL 4	007, 45 4
−	008, 30
RCL 1	009, 45 1
=	010, 36
g GTO 000	011,43,33,000
RCL 5	012, 45 5
+	013, 40
RCL 1	014, 45 1
÷	015, 10
x≷y	016, 34
=	017, 36
g GTO 000	018,43,33,000

12c platinum / 12C RPN KEYSTROKES	DISPLAY
RCL 4	019, 45 4
÷	020, 10
RCL 2	021, 45 2
+	022, 40
g GTO 000	023,43,33,000
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
RCL 1	019, 45 1
+	020, 40
RCL 5	021, 45 5
÷	022, 10
RCL 4	023, 45 4
+	024, 40
RCL 2	025, 45 2
=	026, 36
g GTO 000	027,43,33,000
f P/R	

	REGISTERS		
n: Unused	I: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: Unused	R_1: *F*	R_2: *V*
R_3: *P*	R_4: *U*	R_5: *GP*	R_6-R_6: Unused

Program Instructions:

1. Key in the program and store the known variables as follows:
   a. Key in the fixed costs, F and press STO 1.
   b. Key in the variable costs per unit, V and press STO 2.
   c. Key in the unit price, P (if known) and press STO 3.
   d. Key in the sales volume, U, in units (if known) and press STO 4.
   e. Key in the gross profit, GP, (if known) and press STO 5.
2. To calculate the sales volume to achieve a desired gross profit:
   a. Store values as shown in 1a, 1b, and 1c.
   b. Key in the desired gross profit (zero for break even) and press STO 5.
   c. **RPN:** Press R/S g GTO 010 R/S to calculate the required volume.
   c. **ALG:** Press R/S g GTO 012 R/S to calculate the required volume.
3. To calculate the gross profit at a given sales volume.
   a. Store values as shown in 1a, 1b, 1c, and 1d.
   b. **RPN:** Press R/S g GTO 005 R/S to calculate gross profit.
   b. **ALG:** Press R/S g GTO 006 R/S to calculate gross profit.
4. To calculate the sales price per unit to achieve a desired gross profit at a specified sales volume:
   a. Store values as shown in 1a, 1b, 1d, and 1e.
   b. **RPN:** Press g GTO 016 R/S to calculate the required sales price.
   b. **ALG:** Press g GTO 019 R/S to calculate the required sales price.

Example 2: A manufacturer of automotive accessories produces rear view mirrors. A new line of mirrors will require fixed costs of $35,000 to produce. Each mirror has a variable cost of $8.25. The price of mirrors is tentatively set at $12.50 each. What volume is needed to break even?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
35000 STO 1	35000 STO 1	**35,000.00**	Fixed cost.
8.25 STO 2	8.25 STO 2	**8.25**	Variable cost.
12.5 STO 3	12.5 STO 3	**12.50**	Sales price.
0 STO 5 R/S g GTO 010 R/S	0 STO 5 R/S g GTO 012 R/S	**0.00** **8,235.29**	Break-even volume is between 8,235 and 8,236 units.

What would be the gross profit if the price is raised to $14.00 and the sales volume is 10,000 units?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
14 STO 3	14 STO 3	**14.00**	Sales price. *F* and *V* are already stored.
10000 STO 4	10000 STO 4	**10,000.00**	Volume.
R/S g GTO 005 R/S	R/S g GTO 006 R/S	**22,500.00**	Gross Profit.

Operating Leverage

The degree of operating leverage (*OL*) at a point is defined as the ratio of the percentage change in net operating income to the percentage change in units sold. The greatest degree of operating leverage is found near the break-even point where a small change in sales may produce a very large increase in profits. Likewise, firms with a small degree of operating leverage are operating farther form the break-even point, and they are relatively insensitive to changes in sales volume.

The necessary inputs to calculate the degree of operating leverage and fixed costs (*F*), sales price per unit (*P*), variable cost per unit (*V*) and number of units (*U*).

The operating leverage may be readily calculated as follows:

RPN Mode:

1. Key in the sales price per unit and press [ENTER].
2. Key in the variable cost per unit and press [−].
3. Key in the number of units and press [X] [ENTER] [ENTER].
4. Key in the fixed cost and press [−] [÷] to obtain the operating leverage.

ALG Mode:

1. Key in the sales price per unit and press [−].
2. Key in the variable cost per unit and press [X].
3. Key in the number of units and press [=].
4. Key in the fixed cost and press [Δ%] [%] [CHS] [1/x] to obtain the operating leverage.

Example 1: For the data given in example 1 of the Break-Even Analysis section, calculate the operating leverage at 2000 units and at 5000 units when the sales price is $13 a copy.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
13 [ENTER]	13 [−]	**13.00**	Price per copy.
6.75 [−]	6.75 [X]	**6.25**	Profit per copy.
2000 [X] [ENTER] [ENTER] 12000 [−] [÷]	2000 [=] 12000 [Δ%] [%] [CHS] [1/x]	**25.00**	Close to break-even point.
13 [ENTER]	13 [−]	**13.00**	Price per copy.
6.75 [−]	6.75 [X]	**6.25**	Profit per copy.
5000 [X] [ENTER] [ENTER] 12000 [−] [÷]	5000 [=] 12000 [Δ%] [%] [CHS] [1/x]	**1.62**	Operating further from the breakeven point and less sensitive to changes in sales volume.

For repeated calculations the following HP 12C Platinum program can be used:

12c platinum / 12C RPN KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
RCL 3	001, 45 3
RCL 2	002, 45 2
−	003, 30
X	004, 20
ENTER	005, 36
ENTER	006, 36
RCL 1	007, 45 1
−	008, 30
÷	009, 10
g GTO 000	010,43,33,000
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
RCL 3	001, 45 3
−	002, 30
RCL 2	003, 45 2
X	004, 20
x≷y	005, 34
=	006, 36
RCL 1	007, 45 1
Δ%	008, 24
%	009, 25
CHS	010, 16
1/x	011, 22
g GTO 000	012,43,33,000
f P/R	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: Unused	R_1: *F*	R_2: *V*
R_3: *P*	R_4-R_8: Unused		

Program Instructions:

1. Key in the program.
2. Key in and store input variables *F*, *V* and *P* as described in the Break-Even Analysis program.
3. Key in the sales volume and press R/S to calculate the operating leverage.
4. To calculate a new operating leverage at a different sales volume, key in the new sales volume and press R/S.

Example 2: For the figures given in example 2 of the Break-Even Analysis section, calculate the operating leverage at a sales volume of 9,000 and 20,000 units if the sales price is $12.50 per unit.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
35000 STO 1	35000 STO 1	**35,000.00**	Fixed costs.
8.25 STO 2	8.25 STO 2	**8.25**	Variable cost.
12.5 STO 3	12.5 STO 3	**12.50**	Sales price.
9000 R/S	9000 R/S	**11.77**	Operating leverage near break-even.
20000 R/S	20000 R/S	**1.70**	Operating leverage further from break-even.

Profit and Loss Analysis

The HP 12C Platinum may be programmed to perform simplified profit and loss analysis using the standard profit income formula and can be used as a dynamic simulator to quickly explore ranges of variables affecting the profitability of a marketing operation.

The program operates with net income return and operating expenses as percentages. Both percentage figures are based on *net* sales price.

It may also be used to simulate a company wide income statement by replacing list price with gross sales and manufacturing cost with cost of goods sold.

Any of the five variables: a) list price, b) discount (as a percentage of list price), c) manufacturing cost, d) operating expense (as a percentage), e) net profit after tax (as a percentage) may be calculated if the other four are known.

Since the tax rage varies from company to company, provision is made for inputting your applicable tax rate. The example problem uses a tax rate of 48%.

12c platinum / 12C RPN KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
RCL 5	001, 45 5
RCL 6	002, 45 6
÷	003, 10
RCL 4	004, 45 4
+	005, 40
CHS	006, 16
RCL 0	007, 45 0
+	008, 40
RCL 0	009, 45 0
÷	010, 10
g GTO 000	011,43,33,000
RCL 3	012, 45 3
RCL 1	013, 45 1
RCL 2	014, 45 2
RCL 0	015, 45 0
÷	016, 10
CHS	017, 16
1	018, 1
+	019, 40
X	020, 20
R/S	021, 31
÷	022, 10
CHS	023, 16
1	024, 1

12c platinum ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
RCL 5	001, 45 5
÷	002, 10
RCL 6	003, 45 6
+	004, 40
RCL 4	005, 45 4
−	006, 30
1	007, 1
x≷y	008, 34
%	009, 25
=	010, 36
g GTO 000	011,43,33,000
RCL 3	012, 45 3
RCL 1	013, 45 1
RCL 2	014, 45 2
x≷y	015, 34
−	016, 30
x≷y	017, 34
%	018, 25
=	019, 36
R/S	020, 31
g GTO 027	021,43,33,027
x≷y	022, 34
÷	023, 10
x≷y	024, 34

12c platinum / 12C RPN KEYSTROKES	DISPLAY
[+]	025, 40
[RCL] 0	026, 45 0
[X]	027, 20
[g] [GTO] 000	028,43,33,000
[÷]	029, 10
[CHS]	030, 16
[RCL] 1	031, 45 1
[+]	032, 40
[RCL] 1	033, 45 1
[÷]	034, 10
[RCL] 0	035, 45 0
[X]	036, 20
[g] [GTO] 000	037,43,33,000
[RCL] 5	038, 45 5
[RCL] 6	039, 45 6
[÷]	040, 10
[−]	041, 30
[g] [GTO] 000	042,43,33,000
[RCL] 4	043, 45 5
[−]	044, 30
[RCL] 6	045, 45 6
[X]	046, 20
[g] [GTO] 000	047,43,33,000
[f] [P/R]	

12c platinum ALG KEYSTROKES	DISPLAY
[=]	025, 36
[RCL] 1	026, 45 1
[x≷y]	027, 34
[Δ%]	028, 24
[CHS]	029, 16
[g] [GTO] 000	030,43,33,000
[RCL] 5	031, 45 5
[÷]	032, 10
[RCL] 6	033, 45 6
[−]	034, 30
[x≷y]	035, 34
[x≷y]	036, 34
[=]	037, 36
[g] [GTO] 000	038,43,33,000
[−]	039, 30
[RCL] 4	040, 45 4
[X]	041, 20
[RCL] 6	042, 45 6
[=]	043, 36
[g] [GTO] 000	044,43,33,000
[f] [P/R]	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: 100	R_1: list price	R_2: % discount
R_3: mfg. cost	R_4: % op. exp.	R_5: % net profit	R_6: 1-% tax
R_7-$R_{.3}$: Unused			

Program Instructions:

1. Key in the program and press [f] CLEAR [REG], then key in 100 and press [STO] 0.

2. **RPN:** Key in 1 and press [ENTER], then key in your appropriate tax rate as a decimal and press [−] [STO] 6.

2. **ALG:** Key in 1 and press [−], then key in your appropriate tax rate as a decimal and press [=] [STO] 6.

3. a. Key in the list price in dollars (if known) and press [STO] 1.
   b. Key in the discount in percent (if known) and press [STO] 2.
   c. Key in the manufacturing cost in dollars (if known) and press [STO] 3.
   d. Key in the operating expense in percent (if known) and press [STO] 4.
   e. Key in the net profit after tax in percent (if known) and press [STO] 5.

4. To calculate list price:
   a. Do steps 2 and 3b, c, d, e above.
   b. **RPN:** Press RCL 3 R/S ÷ 1 g GTO 014 R/S ÷ g GTO 000.
   b. **ALG:** Press RCL 3 R/S x≷y ÷ x≷y = 1 g GTO 014 R/S x≷y ÷ x≷y = g GTO 000.
5. To calculate discount:
   a. Do steps 2 and 3a, c, d, e above.
   b. **RPN:** Press RCL 3 R/S g GTO 029 R/S.
   b. **ALG:** Press RCL 3 R/S g GTO 022 R/S.
6. To calculate manufacturing cost:
   a. Do steps 2 and 3a, b, d, e, above.
   b. **RPN:** Press g GTO 013 R/S g GTO 001 R/S X.
   b. **ALG:** Press g GTO 013 R/S g GTO 001 R/S X x≷y =.
7. To calculate operating expense:
   a. Do steps 2 and 3a, b, c, e, above.
   b. **RPN:** Press g GTO 012 R/S R/S g GTO 038 R/S.
   b. **ALG:** Press g GTO 012 R/S R/S g GTO 031 R/S.
8. To calculate net profit after tax:
   a. Do steps 2 and 3a, b, c, d, above.
   b. **RPN:** Press g GTO 012 R/S R/S g GTO 043 R/S.
   b. **ALG:** Press g GTO 012 R/S R/S g GTO 039 R/S.

Example: What is the net return on an item that is sold for $11.98, discounted through distribution an average of 35% and has a manufacturing cost of $2.50? The standard company operating expense is 32% of net shipping (sales) price and tax rate is 48%.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
f CLEAR REG 100 STO 0 1 ENTER .48 − STO 6	f CLEAR REG 100 STO 0 1 − .48 = STO 6	**100.00** **0.52**	48% tax rate.
11.98 STO 1	11.98 STO 1	**11.98**	List price ($).
35 STO 2	35 STO 2	**35.00**	Discount (%).
2.50 STO 3	2.50 STO 3	**2.50**	Manufacturing cost ($).
32 STO 4	32 STO 4	**32.00**	Operating expenses (%).
g GTO 012 R/S R/S g GTO 043 R/S	g GTO 012 R/S R/S g GTO 039 R/S	**67.90** **18.67**	Net profit (%).

If manufacturing expenses increase to $3.25, what is the effect on net profit?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
3.25 STO 3	3.25 STO 3	**3.25**	Manufacturing cost.
g GTO 012 R/S R/S	g GTO 012 R/S R/S	**58.26**	
g GTO 043 R/S	g GTO 039 R/S	**13.66**	Net profit reduced to 13.66%

If the manufacturing cost is maintained at $3.25, how high could the overhead (operating expense) be before the product begins to lose money?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
0 STO 5	0 STO 5	**0.00**	
g GTO 012 R/S R/S	g GTO 012 R/S R/S	**58.26**	
g GTO 038 R/S	g GTO 031 R/S	**58.26**	Maximum operating expense (%).

At 32% operating expense and $3.25 manufacturing cost, what should the list price be to generate 20% net profit?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
20 STO 5	20 STO 5	**20.00**	
RCL 3 R/S ÷	RCL 3 R/S x≷y ÷ x≷y =	**11.00**	
1 g GTO 014 R/S ÷	1 g GTO 014 R/S x≷y ÷ x≷y =	**16.93**	List price ($).

What reduction in manufacturing cost would achieve the same result without necessitating an increase in list price above $11.98?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
g GTO 013 R/S	g GTO 013 R/S	**7.79**	
g GTO 001 R/S X	g GTO 001 R/S X x≷y =	**2.30**	Manufacturing cost ($).

Securities and Options

After-Tax Yield

The following HP 12C Platinum program calculates the after-tax yield to maturity of a bond held for more than one year. The calculation assumes an actual/actual day basis. For after-tax computations, the interest or coupon payments are considered income, while the difference between the bond's face value and its purchase price is considered capital gains.

12c platinum / 12C RPN KEYSTROKES	DISPLAY		
f P/R			
f CLEAR PRGM	000,		
f CLEAR FIN	001,	42	34
STO 7	002,	44	7
R↓	003,		33
STO 6	004,	44	6
RCL 2	005,	45	2
RCL 1	006,	45	1
−	007,		30
RCL 4	008,	45	4
%	009,		25
RCL 2	010,	45	2
x≷y	011,		34
−	012,		30
EEX	013,		26
2	014,		2
÷	015,		10
STO 0	016,	44	0
RCL 3	017,	45	3
RCL 5	018,	45	5
%	019,		25
−	020,		30
RCL 0	021,	45	0
÷	022,		10
PMT	023,		14
RCL 1	024,	45	1
RCL 0	025,	45	0
÷	026,		10
PV	027,		13
RCL 6	028,	45	6
RCL 7	029,	45	7

12c platinum ALG KEYSTROKES	DISPLAY		
f P/R			
f CLEAR PRGM	000,		
f CLEAR FIN	001,	42	34
STO 7	002,	44	7
R↓	003,		33
STO 6	004,	44	6
RCL 2	005,	45	2
−	006,		30
RCL 1	007,	45	1
−	008,		30
RCL 4	009,	45	4
%	010,		25
+	011,		40
RCL 1	012,	45	1
=	013,		36
%	014,		25
STO 0	015,	44	0
RCL 3	016,	45	3
−	017,		30
RCL 5	018,	45	5
%	019,		25
÷	020,		10
RCL 0	021,	45	0
PMT	022,		14
RCL 1	023,	45	1
÷	024,		10
RCL 0	025,	45	0
PV	026,		13
RCL 6	027,	45	6
RCL 7	028,	45	7
f YTM	029,	42	22

12c platinum / 12C RPN KEYSTROKES	DISPLAY
f YTM	030, 42 22
g GTO 000	031,43,33,000
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
g GTO 000	030,43,33,000
f P/R	

	REGISTERS		
n: Unused	i: Yield	PV: Used	PMT: Used
FV: 0	R_0: Used	R_1: Purchase price	R_2: Sales price
R_3: Coupon rate	R_4: Capital rate	R_5: Income rate	R_6: Used
R_7: Used	R_8-$R_{.5}$: Unused		

Program Instructions:

1. Key in the program.
2. Key in the purchase price and press STO 1.
3. Key in the sales price and press STO 2.
4. Key in the annual coupon rate (as a percentage) and press STO 3.
5. Key in capital gains tax rate (as a percentage) and press STO 4.
6. Key in the income tax rate (as a percentage) and press STO 5.
7. Press g M.DY.
8. Key in the purchase date (MM.DDYYYY) and press ENTER (=).
9. Key in the assumed sell date (MM.DDYYYY) and press R/S to find the after-tax yield (as a percentage).
10. For the same bond but different date return to step 8.
11. For a new case return to step 2.

Example: You can buy a 7% bond on October 1, 2003 for $70 and expect to sell it in 5 years for $90. What is your net (after-tax) yield over the 5-year period if interim coupon payments are considered as income, and your tax bracket is 50%?

(One-half of the long term capital gain is taxable at 50%, so the tax on capital gains alone is 25%)

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
70 STO 1	70 STO 1	**70.00**	Purchase price.
90 STO 2	90 STO 2	**90.00**	Selling price.
7 STO 3	7 STO 3	**7.00**	Annual coupon rate.
25 STO 4	25 STO 4	**25.00**	Capital gains tax rate.
50 STO 5	50 STO 5	**50.00**	Income tax rate.
g M.DY	g M.DY		
10.012003 ENTER	10.012003 =	**10.01**	Purchase Date.
10.012008 R/S	10.012008 R/S	**8.53**	% after tax yield.

Discounted Notes

A note is a written agreement to pay a sum of money plus interest at a certain rate. Notes do not have periodic coupons, since all interest is paid at maturity.

A discounted note is a note that is purchased below its face value. The following HP 12C Platinum program finds the price and/or yield* of a discounted note.

12c platinum / 12C RPN KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
RCL 1	001, 45 1
RCL 2	002, 45 2
g ΔDYS	003, 43 26
RCL 3	004, 45 3
÷	005, 10
RCL 5	006, 45 5
%	007, 25
1	008, 1
x≷y	009, 34
−	010, 30
RCL 4	011, 45 4
X	012, 20
STO 5	013, 44 5
R/S	014, 31
RCL 1	015, 45 1
RCL 2	016, 45 2
g ΔDYS	017, 43 26
RCL 3	018, 45 3
x≷y	019, 34
÷	020, 10
RCL 4	021, 45 4
RCL 5	022, 45 5
÷	023, 10
1	024, 1
−	025, 30
X	026, 20
EEX	027, 26
2	028, 2
X	029, 20
g GTO 000	030,43,33,000
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
RCL 1	001, 45 1
RCL 2	002, 45 2
g ΔDYS	003, 43 26
÷	004, 10
RCL 3	005, 45 3
X	006, 20
RCL 5	007, 45 5
=	008, 36
RCL 4	009, 45 4
−	010, 30
x≷y	011, 34
%	012, 25
=	013, 36
STO 5	014, 44 5
R/S	015, 31
RCL 1	016, 45 1
RCL 2	017, 45 2
g ΔDYS	018, 43 26
RCL 5	019, 45 5
RCL 4	020, 45 4
Δ%	021, 24
x≷y	022, 34
R↓	023, 33
÷	024, 10
x≷y	025, 34
X	026, 20
RCL 3	027, 45 3
=	028, 36
g GTO 000	029,43,33,000
f P/R	

* The yield is a reflection of the return on an investment.

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: Unused	R_1: Settl. date	R_2: Mat. Date
R_3: 360 or 360	R_4: redemp. Value	R_5: dis./price	R_6-$R_{.5}$: Unused

Program Instructions:

1. Key in the program.
2. Press [g] [M.DY].
3. Key in the settlement date (MM.DDYYYY) and press [STO] 1.
4. Key in the maturity date (MM.DDYYYY) and press [STO] 2.
5. Key in the number of days in a year (360 or 365) and press [STO] 3.
6. Key in the redemption value per $100 and press [STO] 4.
7. To calculate the purchase price:
   a. Key in the discount rate and press [STO] 5.
   b. Press [R/S] to calculate the purchase price.
   c. Press [R/S] to calculate the yield.
   d. For a new case, go to step 3.
8. To calculate the yield when the price is known:
   a. Key in the price and press [STO] 5.
   b. **RPN:** Press [g] [GTO] 015 [R/S] to calculate the yield.
   b. **ALG:** Press [g] [GTO] 016 [R/S] to calculate the yield.
   c. For a new case, go to step 3.

Example 1: Calculate the price and yield on this bill: settlement date October 8, 2002; maturity date March 21, 2003; discount rate 7.80%. Compute on a 360 day basis.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
[g] [M.DY]	[g] [M.DY]		
10.082002 [STO] 1	10.082002 [STO] 1	**10.08**	Settlement date.
3.212003 [STO] 2	3.212003 [STO] 2	**3.21**	Maturity date.
360 [STO] 3	360 [STO] 3	**360.00**	360 day basis.
100 [STO] 4	100 [STO] 4	**100.00**	Redemption value per $100.
7.8 [STO] 5	7.8 [STO] 5	**7.80**	Discount rate.
[R/S]	[R/S]	**96.45**	Price.
[R/S]	[R/S]	**8.09**	Yield.

Example 2: Determine the yield of this security; settlement date June 25, 2002; maturity date September 10, 2002; price $99.45; redemption value $101.33. Assume 360 day basis.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
6.252002 STO 1	6.252002 STO 1	**6.25**	Settlement date.
9.102002 STO 2	9.102002 STO 2	**9.10**	Maturity date.
360 STO 3	360 STO 3	**360.00**	360 day basis.
101.33 STO 4	101.33 STO 4	**101.33**	Redemption value per $100.
99.45 STO 5	99.45 STO 5	**99.45**	Price.
g GTO 015 R/S	g GTO 016 R/S	**8.84**	Yield.

Black-Scholes Formula for Valuing European Options

This program implements the Black-Scholes formula which has been used extensively in option markets worldwide since its publication in the early 1970's. The five inputs are simply keyed into the five financial variables and then R/S displays the call option value, and x≷y shows the put option value. The option values produced are accurate to at least the nearest cent for asset and strike prices under $100.

Reference: Hutchins, 2003, *Black-Scholes takes over the HP12C*, HPCC (www.hpcc.org) DataFile,V22,N3 pp13-21.

12c platinum / 12C RPN KEYSTROKES	DISPLAY		
f P/R			
f CLEAR PRGM	000,		
RCL n	001,	45	11
RCL i	002,	45	12
%	003,		25
CHS	004,		16
g e^x	005,	43	22
RCL FV	006,	45	15
X	007,		20
STO 4	008,	44	4
x≷y	009,		34
g $\sqrt{x}$	010,	43	21
RCL PMT	011,	45	14
%	012,		25
STO 3	013,	44	3
RCL PV	014,	45	13
RCL 4	015,	45	4
÷	016,		10

12c platinum ALG KEYSTROKES	DISPLAY		
f P/R			
f CLEAR PRGM	000,		
RCL n	001,	45	11
X	002,		20
RCL i	003,	45	12
%	004,		25
=	005,		36
CHS	006,		16
g e^x	007,	43	22
X	008,		20
RCL FV	009,	45	15
=	010,		36
STO 4	011,	44	4
RCL n	012,	45	11
g $\sqrt{x}$	013,	43	21
X	014,		20
RCL PMT	015,	45	14
%	016,		25

12c platinum / 12C RPN KEYSTROKES	DISPLAY		
g LN	017,	43	23
x≷y	018,		34
÷	019,		10
g LSTx	020,	43	40
2	021,		2
STO 5	022,	44	5
÷	023,		10
+	024,		40
STO 6	025,	44	6
RCL 3	026,	45	3
−	027,		30
STO 3	028,	44	3
ENTER	029,		36
X	030,		20
g √x	031,	43	21
g LSTx	032,	43	40
2	033,		2
÷	034,		10
CHS	035,		16
g e^x	036,	43	22
x≷y	037,		34
3	038,		3
•	039,		48
0	040,		0
0	041,		0
6	042,		6
÷	043,		10
1	044,		1
+	045,		40
1/x	046,		22
X	047,		20
g LSTx	048,	43	40
g LSTx	049,	43	40
1	050,		1
8	051,		8
7	052,		7
X	053,		20
2	054,		2
4	055,		4
−	056,		30
X	057,		20
8	058,		8
7	059,		7
+	060,		40

12c platinum ALG KEYSTROKES	DISPLAY		
=	017,		36
STO 3	018,	44	3
RCL PV	019,	45	13
÷	020,		10
RCL 4	021,	45	4
=	022,		36
g LN	023,	43	23
÷	024,		10
x≷y	025,		34
=	026,		36
RCL 3	027,	45	3
÷	028,		10
2	029,		2
STO 5	030,	44	5
+	031,		40
x≷y	032,		34
−	033,		30
STO 6	034,	44	6
RCL 3	035,	45	3
=	036,		36
STO 3	037,	44	3
g x^2	038,	43	20
÷	039,		10
2	040,		2
=	041,		36
CHS	042,		16
g e^x	043,	43	22
RCL 3	044,	45	3
g x^2	045,	43	20
g √x	046,	43	21
÷	047,		10
3	048,		3
•	049,		48
0	050,		0
0	051,		0
6	052,		6
+	053,		40
1	054,		1
=	055,		36
1/x	056,		22
STO 2	057,	44	2
X	058,		20
x≷y	059,		34
=	060,		36

12c platinum / 12C RPN KEYSTROKES	DISPLAY
X	061, 20
•	062, 48
2	063, 2
%	064, 25
RCL 3	065, 45 3
x≷y	066, 34
STO 3	067, 44 3
CLx	068, 35
x≷y	069, 34
g x≤y	070, 43 34
g GTO 077	071,43,33,077
1	072, 1
STO − 3	073,44 30 3
CHS	074, 16
STO X 3	075,44 20 3
x≷y	076, 34
RCL 5	077, 45 5
g x=0	078, 43 35
g GTO 089	079,43,33,089
RCL 6	080, 45 6
RCL 3	081, 45 3
RCL 4	082, 45 4
X	083, 20
STO 6	084, 44 6
CLx	085, 35
STO 5	086, 44 5
x≷y	087, 34
g GTO 028	088,43,33,028
x≷y	089, 34
RCL 3	090, 45 3
RCL PV	091, 45 13
STO − 4	092,44 30 4
X	093, 20
RCL 6	094, 45 6
−	095, 30
STO + 4	096,44 40 4
RCL 4	097, 45 4
x≷y	098, 34
STO 5	099, 44 5
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
1	061, 1
8	062, 8
7	063, 7
X	064, 20
RCL 2	065, 45 2
−	066, 30
2	067, 2
4	068, 4
X	069, 20
RCL 2	070, 45 2
+	071, 40
8	072, 8
7	073, 7
X	074, 20
x≷y	075, 34
X	076, 20
•	077, 48
2	078, 2
%	079, 25
=	080, 36
RCL 3	081, 45 3
x≷y	082, 34
STO 3	083, 44 3
CLx	084, 35
x≷y	085, 34
g x≤y	086, 43 34
g GTO 093	087,43,33,093
1	088, 1
STO − 3	089,44 30 3
CHS	090, 16
STO X 3	091,44 20 3
x≷y	092, 34
RCL 5	093, 45 5
g x=0	094, 43 35
g GTO 106	095,43,33,106
RCL 6	096, 45 6
RCL 3	097, 45 3
X	098, 20
RCL 4	099, 45 4
=	100, 36

12c platinum / 12C RPN KEYSTROKES	DISPLAY

12c platinum ALG KEYSTROKES	DISPLAY
STO 6	101, 44 6
CLx	102, 35
STO 5	103, 44 5
x≷y	104, 34
g GTO 037	105,43,33,037
x≷y	106, 34
RCL PV	107, 45 13
STO − 4	108,44 30 4
X	109, 20
RCL 3	110, 45 3
−	111, 30
RCL 6	112, 45 6
=	113, 36
STO + 4	114,44 40 4
RCL 4	115, 45 4
x≷y	116, 34
STO 5	117, 44 5
g GTO 000	118,43,33,000
f P/R	

	REGISTERS		
n: Term to expiry	i: Interest rate (%)	PV: Stock price	PMT: Volatility (%)
FV: Strike price	R_0: Unused	R_1: Unused	R_2: Unused
R_3: $N(d_1)$	R_4: Put value	R_5: Call value	R_6: $Q \cdot N(d_2)$
R_7-R_9: Unused			

Note: The n, i and PMT values must all be based on the same *time unit* (for example: n is measured in years or months and i and PMT are rates per year or per month). i is a continuous percentage rate. PMT is the standard deviation of the continuous percentage stock return (as observed over the *time unit*). For sensible output, all inputs should be positive. The PMT=0 case can be simulated by using a PMT arbitrarily close to 0.

Program Instructions

1. Key in the program.
2. Enter the five inputs into the five financial registers. These values are preserved by the program.
   a. Key in the unexpired term of the option and press n.
   b. Key in the risk-free interest rate as a percentage and press i.
   c. Key in the current (or spot) stock price and press PV.
   d. Key in the volatility assumption as a percentage and press PMT.
   e. Key in the strike price and press FV.
3. Press R/S. The Call value is displayed. Press x≷y to see the Put value.

Example 1: An option has 6 months to run and a strike price of $45. Find Call and Put values assuming a spot price of $52, return volatility of 20.54% per month and a risk-free interest rate of ½% per month. Show how to re-scale n, i and PMT to use a yearly time unit, and how to re-scale them back again to the original monthly basis.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
6 n	6 n	**6.00**	Time to expiry (months).
.5 i	.5 i	**0.50**	Interest rate (% per month).
52 PV	52 PV	**52.00**	Stock price.
20.54 PMT	20.54 PMT	**20.54**	Volatility (% per month).
45 FV	45 FV	**45.00**	Strike price.
R/S	R/S	**14.22**	Call value.
x≷y	x≷y	**5.89**	Put value.
RCL g 12x n	RCL g 12x n	**0.50**	Years to expiry.
RCL g 12÷ i	RCL g 12÷ i	**6.00**	Yearly interest rate %.
RCL PMT 12 g √x X PMT	RCL PMT X 12 g √x PMT	**71.15**	Yearly volatility %.
R/S	R/S	**14.22**	Call value (unchanged).
RCL n g 12x	RCL n g 12x	**6.00**	Months to expiry.
RCL i g 12÷	RCL i g 12÷	**0.50**	Monthly interest rate %.
RCL PMT 12 g √x ÷ PMT	RCL PMT ÷ 12 g √x PMT	**20.54**	Monthly volatility %.

The next example is *Example 12.7* from *Options, Futures, and Other Derivatives (5th Edition)* by John C. Hull (Prentice Hall, 2002).

Example 2: The stock price six months from the expiration of an option is $42, the exercise price of the option is $40, the risk-free interest rate is 10% per annum, and the volatility is 20% per annum. Find Call and Put values.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
.5 n	.5 n	**0.50**	Time to expiry (years).
10 i	10 i	**10.00**	Interest rate (% per year).
42 PV	42 PV	**42.00**	Stock price.
20 PMT	20 PMT	**20.00**	Volatility (% per year).
40 FV	40 FV	**40.00**	Strike price.
R/S	R/S	**4.76**	Call value.
x≷y	x≷y	**0.81**	Put value.

Forecasting

Simple Moving Average

Moving averages are often useful in recording of forecasting sales figures, expenses or manufacturing volume. There are many different types of moving average calculations. An often used, straightforward method of calculation is presented here.

In a moving average a specified number of data points are averaged. When there is a new piece of input data, the oldest piece of data is discarded to make room for the latest input. This replacement scheme makes the moving average a valuable tool in following trends. The fewer the number of data points, the more trend sensitive the average becomes. With a large number of data points, the average behaves more like a regular average, responding slowly to new input data.

A simple moving average may be calculated with your HP 12C Platinum as follows.

1. Press [f] CLEAR [REG].
2. Key in the first *m* data points (where *m* is the number of data points in the average) and press [Σ+] after each entry.
3. Press [g] [x̄] to obtain the first average.
4. Key in the oldest (first value) entered in step 2 and press [g] [Σ−].
5. Key in the newest data point (*m* + 1) and press [Σ+].
6. Press [g] [x̄] to obtain the next value of the moving average.
7. Repeat steps 4 through 5 for the remaining data.

Example 1: An electronics sales firm wished to calculate a 3-month moving average for the dollar volume of components sold each month. Sales for the first six months of this year were:

January	$211,570	April	131,760
February	112,550	May	300,500
March	190,060	June	271,120

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
[f] CLEAR [REG]	[f] CLEAR [REG]	**0.00**	
211570 [Σ+]	211570 [Σ+]	**1.00**	
112550 [Σ+]	112550 [Σ+]	**2.00**	
190060 [Σ+]	190060 [Σ+]	**3.00**	
[g] [x̄]	[g] [x̄]	**171,393.33**	3-month average for March.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
211570 g Σ− 131760 Σ+ g x̄	211570 g Σ− 131760 Σ+ g x̄	**2.00** **3.00** **144,790.00**	3-month average for April.
112550 g Σ− 300500 Σ+ g x̄	112550 g Σ− 300500 Σ+ g x̄	**2.00** **3.00** **207,440.00**	3-month average for May.
190060 g Σ− 271120 Σ+ g x̄	190060 g Σ− 271120 Σ+ g x̄	**2.00** **3.00** **234,460.00**	3-month average for June.

For repeated calculations the following HP 12C Platinum program can be used for up to a 12 element moving average:

12c platinum / 12C RPN KEYSTROKES	DISPLAY	12c platinum ALG KEYSTROKES	DISPLAY
f P/R		f P/R	
f CLEAR PRGM	000,	f CLEAR PRGM	000,
RCL 1	001, 45 1	RCL 1	001, 45 1
RCL 2	002, 45 2	+	002, 40
STO 1	003, 44 1	RCL 2	003, 45 2
+	004, 40	STO 1	004, 44 1
RCL 3	005, 45 3	+	005, 40
STO 2	006, 44 2	RCL 3	006, 45 3
+[3]	007, 40	STO 2[3]	007, 44 2
RCL 4	008, 45 4	+	008, 40
STO 3	009, 44 3	RCL 4	009, 45 4
+[4]	010, 40	STO 3[4]	010, 44 3
RCL 5	011, 45 5	+	011, 40
STO 4	012, 44 4	RCL 5	012, 45 5
+[5]	013, 40	STO 4[5]	013, 44 4
RCL 6	014, 45 6	+	014, 40
STO 5	015, 44 5	RCL 6	015, 45 6
+[6]	016, 40	STO 5[6]	016, 44 5
RCL 7	017, 45 7	+	017, 40
STO 6	018, 44 6	RCL 7	018, 45 7
+[7]	019, 40	STO 6[7]	019, 44 6
RCL 8	020, 45 8	+	020, 40
STO 7	021, 44 7	RCL 8	021, 45 8
+[8]	022, 40	STO 7[8]	022, 44 7
RCL 9	023, 45 9	+	023, 40
STO	024, 44 8	RCL 9	024, 45 9
+[9]	025, 40	STO 8[9]	025, 44 8
RCL • 0	026,45 48 0	+	026, 40

12c platinum / 12C RPN KEYSTROKES	DISPLAY
STO 9	027, 44 9
+ [10]	028, 40
RCL • 1	029,45 48 1
STO • 0	030,44 48 0
+ [11]	031, 40
RCL • 2	032,45 48 2
STO • 1	033,44 48 1
+ [12]	034, 40
RCL 0	035, 45 0
÷	036, 10
R/S	037, 31
STO m*	038, 44 --
g GTO 001	039,43,33,001
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
RCL • 0	027,45 48 0
STO 9 [10]	028, 44 9
+	029, 40
RCL • 1	030,45 48 1
STO • 0 [11]	031,44 48 0
+	032, 40
RCL • 2	033,45 48 2
STO • 1 [12]	034,44 48 1
÷	035, 10
RCL 0	036, 45 0
=	037, 36
R/S	038, 31
STO m*	039, 44 --
g GTO 001	040,43,33,001
f P/R	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: *m*	R_1: X_1	R_2: X_2
R_3: X_3	R_4: X_4	R_5: X_5	R_6: X_6
R_7: X_7	R_8: X_8	R_9: X_9	R_0: X_0
R_1: X_{11}	R_2: X_{12}	R_3-R_4: Unused	

This program can be used for a moving average of 2 to 12 elements. It may be shortened considerably for moving averages with less than 12 elements. To do this, key in the program, as shown, from line 01 until you reach a + (ALG: a STO command) superscripted with the number of elements you desire. Key in this line, then skip the rest of the program down to line 35. Then key in lines 035 through 039 (ALG:040), being sure to specify the register number at line 038 (ALG:039), STO *m*, corresponding to the number of elements you are using. (For instance, for a 5 element moving average, key in lines 01 through 13 then go to line 35 in the listing and key in the balance of the program. Obviously the program listing line 38 (ALG:039), STO *m* becomes the displayed line 017, STO 5 in RPN and 018, STO 5 in ALG).

* At step 038 (ALG:039), *m*=number of elements in the moving average, i.e. for a 5 element moving average line 038 (ALG:039) would be STO 5 and for a 12 element average line 38 (ALG:039) would be STO • 2

Program Instructions:

1. Key in the program.
2. Press f CLEAR REG. Key in the number of elements, *m*, and press STO 0.
3. Key in the first data point and press STO 1.
4. Key in the second data point and press STO 2.
5. Continue as above, keying in and storing each data point in its appropriate register until *m* data points have been stored.
6. Press g GTO 000 R/S to calculate the first moving average.
7. Key in the next data point and press R/S to calculate the next moving average.
8. Repeat step 7 for each new data point.

Example 2: Calculate the 3-element moving average for the data given in example 1. Your modified program listing will look like this:

12c platinum / 12C RPN KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
RCL 1	001, 45 1
RCL 2	002, 45 2
STO 1	003, 44 1
+	004, 40
RCL 3	005, 45 3
STO 2	006, 44 2
+	007, 40
RCL 0	008, 45 0
÷	009, 10
R/S	010, 31
STO 3	011, 44 3
g GTO 001	012,43,33,001
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
RCL 1	001, 45 1
+	002, 40
RCL 2	003, 45 2
STO 1	004, 44 1
+	005, 40
RCL 3	006, 45 3
STO 2	007, 44 2
÷	008, 10
RCL 0	009, 45 0
=	010, 36
R/S	011, 31
STO 3	012, 44 3
g GTO 001	013,43,33,001
f P/R	

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
f CLEAR REG	f CLEAR REG	**0.00**	
3 STO 0	3 STO 0	**3.00**	
211570 STO 1	211570 STO 1	**211,570.00**	
112550 STO 2	112550 STO 2	**112,550.00**	
190060 STO 3	190060 STO 3	**190,060.00**	
g GTO 000 R/S	g GTO 000 R/S	**171,393.33**	3-month average for March.
131760 R/S	131760 R/S	**144,790.00**	3-month average for April.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
300500 R/S	300500 R/S	**207,440.00**	3-month average for May.
271120 R/S	271120 R/S	**234,460.00**	3-month average for June.

Seasonal Variation Factors Based on Centered Moving Averages

Seasonal variation factors are useful concepts in many types of forecasting. There are several methods of developing seasonal moving averages, on the of more common ways being to calculate them as a ratio of the periodic value to a centered moving average for the same period.

For instance, to determine the sales for the 3rd quarter of a given year a centered moving average for that quarter would be calculated from sales figures from the 1st, 2nd, 3rd and 4th quarters of the year and the 1st quarter of the following year. The seasonal variation factor for that 3rd quarter would then be the ratio of the actual sales in the 3rd quarter to the centered moving average for that quarter.

While quarterly seasonal variations are commonly used, the HP 12C Platinum can also be programmed to calculate monthly seasonal variations using a centered 12 month moving average. Programs for both of these calculations are represented here:

An HP 12C Platinum program to calculate the quarterly seasonal variations based on a centered 4-point moving average is:

12c platinum / 12C RPN KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
RCL 1	001, 45 1
2	002, 2
÷	003, 10
RCL 2	004, 45 2
STO 1	005, 44 1
+	006, 40
RCL 3	007, 45 3
STO 2	008, 44 2
+	009, 40
RCL 4	010, 45 4
STO 3	011, 44 3
+	012, 40
RCL 5	013, 45 5
STO 4	014, 44 4
2	015, 2
÷	016, 10
+	017, 40

12c platinum ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
RCL 1	001, 45 1
÷	002, 10
2	003, 2
+	004, 40
RCL 2	005, 45 2
STO 1	006, 44 1
+	007, 40
RCL 3	008, 45 3
STO 2	009, 44 2
+	010, 40
RCL 4	011, 45 4
STO 3	012, 44 3
=	013, 36
RCL 5	014, 45 5
STO 4	015, 44 4
÷	016, 10
2	017, 2

12c platinum / 12C RPN KEYSTROKES	DISPLAY
4	018, 4
÷	019, 10
R/S	020, 31
RCL 2	021, 45 2
%T	022, 23
R/S	023, 31
STO 5	024, 44 5
g GTO 001	025,43,33,001
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
+	018, 40
x≷y	019, 34
÷	020, 10
4	021, 4
=	022, 36
R/S	023, 31
RCL 2	024, 45 2
%T	025, 23
R/S	026, 31
STO 5	027, 44 5
g GTO 001	028,43,33,001
f P/R	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: n	R_1: X_1	R_2: X_2
R_3: X_3	R_4: X_4	R_5: X_5	R_6-R_6: Unused

Program Instructions:

1. Key in the program.
2. Press f CLEAR REG.
3. Key in the quarterly sales figures starting with the first quarter:
   a. Key in 1st quarter sales and press STO 1.
   b. Key in 2nd quarter sales and press STO 2.
   c. Key in 3rd quarter sales and press STO 3.
   d. Key in 4th quarter sales and press STO 4.
   e. Key in the 1st quarter sales for the next year and press STO 5.
4. Press g GTO 000 R/S to calculate the centered moving average for the 3rd quarter of the first year.
5. Press R/S to calculate the seasonal variation for this quarter.
6. Key in the next quarter's sales and press R/S to calculate the moving average for the next quarter.
7. Press R/S to calculate the seasonal variation.
8. Repeat steps 6 and 7 for the balance of the data.

Example: Econo-Wise Home Appliance Company had quarterly sales for the years 2000 thru 2002 as follows:

Sales (IN $K)				
Quarterly	**1st**	**2nd**	**3rd**	**4th**
2000	397	376	460	501
2001	455	390	530	560
2002	513	434	562	593

Find the centered 4-quarter moving average and seasonal variation factor for each quarter.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
f CLEAR REG	f CLEAR REG	**0.00**	
397 STO 1	397 STO 1	**397.00**	
376 STO 2	376 STO 2	**376.00**	
460 STO 3	460 STO 3	**460.00**	
501 STO 4	501 STO 4	**501.00**	
455 STO 5	455 STO 5	**455.00**	
g GTO 000 R/S	g GTO 000 R/S	**440.75**	Centered 4-element average for 3rd quarter, 2000.
R/S	R/S	**104.37**	Seasonal variation factor.
390 R/S	390 R/S	**449.75**	4th quarter, 2000.
R/S	R/S	**111.40**	
530 R/S	530 R/S	**460.25**	1st quarter, 2001.
R/S	R/S	**98.86**	
560 R/S	560 R/S	**476.38**	2nd quarter, 2001.
R/S	R/S	**81.87**	
513 R/S	513 R/S	**491.00**	3rd quarter, 2001.
R/S	R/S	**107.94**	
434 R/S	434 R/S	**503.75**	4th quarter, 2001.
R/S	R/S	**111.17**	
562 R/S	562 R/S	**513.25**	1st quarter, 2002.
R/S	R/S	**99.95**	
593 R/S	593 R/S	**521.38**	2nd quarter, 2002.
R/S	R/S	**83.24**	

Now, what is the average of each quarter's seasonal variation for the two years?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
f CLEAR Σ	f CLEAR Σ	**0.00**	
98.86 Σ+	98.86 Σ+	**1.00**	
99.95 Σ+	99.95 Σ+	**2.00**	
g x̄	g x̄	**99.41**	1st quarter average seasonal variation, %.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
f CLEAR Σ 81.87 Σ+ 83.24 Σ+ g x̄	f CLEAR Σ 81.87 Σ+ 83.24 Σ+ g x̄	**0.00** **1.00** **2.00** **82.56**	2nd quarter average seasonal variation, %.
f CLEAR Σ 104.37 Σ+ 107.94 Σ+ g x̄	f CLEAR Σ 104.37 Σ+ 107.94 Σ+ g x̄	**0.00** **1.00** **2.00** **106.16**	3rd quarter average seasonal variation, %.
f CLEAR Σ 111.4 Σ+ 111.17 Σ+ g x̄	f CLEAR Σ 111.4 Σ+ 111.17 Σ+ g x̄	**0.00** **1.00** **2.00** **111.29**	4th quarter average seasonal variation, %.

An HP 12C Platinum program to calculate a centered 12 month moving average and seasonal variation factor is as follows:

12c platinum / 12C RPN KEYSTROKES	DISPLAY		
f P/R			
f CLEAR PRGM	000,		
RCL 1	001,	45	1
2	002,		2
÷	003,		10
RCL 2	004,	45	2
STO 1	005,	44	1
+	006,		40
RCL 3	007,	45	3
STO 2	008,	44	2
+	009,		40
RCL 4	010,	45	4
STO 3	011,	44	3
+	012,		40
RCL 5	013,	45	5
STO 4	014,	44	4
+	015,		40
RCL 6	016,	45	6
STO 5	017,	44	5
+	018,		40
RCL 7	019,	45	7
STO 6	020,	44	6

12c platinum ALG KEYSTROKES	DISPLAY		
f P/R			
f CLEAR PRGM	000,		
RCL 1	001,	45	1
÷	002,		10
2	003,		2
+	004,		40
RCL 2	005,	45	2
STO 1	006,	44	1
+	007,		40
RCL 3	008,	45	3
STO 2	009,	44	2
+	010,		40
RCL 4	011,	45	4
STO 3	012,	44	3
+	013,		40
RCL 5	014,	45	5
STO 4	015,	44	4
+	016,		40
RCL 6	017,	45	6
STO 5	018,	44	5
+	019,		40
RCL 7	020,	45	7

12c platinum / 12C RPN KEYSTROKES	DISPLAY
[+]	021, 40
[RCL] 8	022, 45 8
[STO] 7	023, 44 7
[+]	024, 40
[RCL] 9	025, 45 9
[STO] 8	026, 44 8
[+]	027, 40
[RCL] [•] 0	028,45 48 0
[STO] 9	029, 44 9
[+]	030, 40
[RCL] [•] 1	031,45 48 1
[STO] [•] 0	032,44 48 0
[+]	033, 40
[RCL] [•] 2	034,45 48 2
[STO] [•] 1	035,44 48 1
[+]	036, 40
[RCL] [•] 3	037,45 48 3
[STO] [•] 2	038,44 48 2
2	039, 2
[÷]	040, 10
[+]	041, 40
[RCL] 0	042, 45 0
[÷]	043, 10
[R/S]	044, 31
[RCL] 6	045, 45 6
[%T]	046, 23
[R/S]	047, 31
[STO] [•] 3	048,44 48 3
[g] [GTO] 001	048,43,33,001
[f] [P/R]	

12c platinum ALG KEYSTROKES	DISPLAY
[STO] 6	021, 44 6
[+]	022, 40
[RCL] 8	023, 45 8
[STO] 7	024, 44 7
[+]	025, 40
[RCL] 9	026, 45 9
[STO] 8	027, 44 8
[+]	028, 40
[RCL] [•] 0	029,45 48 0
[STO] 9	030, 44 9
[+]	031, 40
[RCL] [•] 1	032,45 48 1
[STO] [•] 0	033,44 48 0
[+]	034, 40
[RCL] [•] 2	035,45 48 2
[STO] [•] 1	036,44 48 1
[=]	037, 36
[RCL] [•] 3	038,45 48 3
[STO] [•] 2	039,44 48 2
[÷]	040, 10
2	041, 2
[+]	042, 40
[x≷y]	043, 34
[÷]	044, 10
[RCL] 0	045, 45 0
[=]	046, 36
[R/S]	047, 31
[RCL] 6	048, 45 6
[%T]	049, 23
[R/S]	050, 31
[STO] [•] 3	051,44 48 3
[g] [GTO] 001	052,43,33,001
[f] [P/R]	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: n	R_1: X_1	R_2: X_2
R_3: X_3	R_4: X_4	R_5: X_5	R_6: X_6
R_7: X_7	R_8: X_8	R_9: X_9	$R_{.0}$: X_{10}
$R_{.1}$: X_{11}	$R_{.2}$: X_{12}	$R_{.3}$: X_{13}	

Program Instructions:

1. Key in the program.
2. Press [f] CLEAR [REG].
3. Key in 12 and press [STO] 0.
4. Key in the values for the first 13 months, storing them one at a time in registers 1 through .3; i.e.
   Key in the 1st month and press [STO] 1.
   Key in the 2nd month and press [STO] 2, etc.,
   Key in the 10th month and press [STO] [•] 0, etc.,
   Key in the 13th month and press [STO] [•] 3.
5. Press [g] [GTO] 000 [R/S] to calculate the centered moving average for the 7th month.
6. Press [R/S] to calculate the seasonal variation for that month.
7. Key in the value for the next month (14th) and press [R/S] to calculate the moving average for the next month (8th).
8. Repeat steps 6 and 7 for the balance of the data.

These programs may be customized by the user for different types of centered moving averages. Inspection of the programs will show how they can be modified.

Gompertz Curve Trend Analysis

A useful curve for evaluating sales trends, etc., is the Gompertz curve. This is a "growth" curve having a general "S" shape and may be used to describe series of data where the early rate of growth is small, then accelerates for a period of time and then slows again as the time grows long. The sales curve for many products follow this trend during the introductory, growth and maturity phases.

The data points to be fit to a Gompertz curve should be equally spaced along the x (or time) axis and all the data points must be positive. The points are divided serially into 3 groups for data entry.

The following HP 12C Platinum program processes the data, fits it to a Gompertz curve and calculates estimated values for future data points. The 3 constants which characterize the curve are available to the user if desired.

12c platinum / 12C RPN KEYSTROKES	DISPLAY
[f] [P/R]	
[f] CLEAR [PRGM]	000,
[g] [LN]	001, 43 23
[STO] [+] 3	002,44 40 3
[R↓]	003, 33

12c platinum ALG KEYSTROKES	DISPLAY
[f] [P/R]	
[f] CLEAR [PRGM]	000,
[g] [LN]	001, 43 23
[STO] [+] 3	002,44 40 3
[R↓]	003, 33

12c platinum / 12C RPN KEYSTROKES	DISPLAY	12c platinum ALG KEYSTROKES	DISPLAY
g LN	004, 43 23	g LN	004, 43 23
STO + 2	005,44 40 2	STO + 2	005,44 40 2
R↓	006, 33	R↓	006, 33
g LN	007, 43 23	g LN	007, 43 23
STO + 1	008,44 40 1	STO + 1	008,44 40 1
1	009, 1	1	009, 1
STO + 4	010,44 40 4	STO + 4	010,44 40 4
RCL 4	011, 45 4	RCL 4	011, 45 4
g GTO 000	012,43,33,000	g GTO 000	012,43,33,000
RCL 3	013, 45 3	RCL 2	013, 45 2
RCL 2	014, 45 2	−	014, 30
−	015, 30	RCL 1	015, 45 1
RCL 2	016, 45 2	=	016, 36
RCL 1	017, 45 1	STO 8	017, 44 8
−	018, 30	RCL 3	018, 45 3
÷	019, 10	−	019, 30
RCL 4	020, 45 4	RCL 2	020, 45 2
1/x	021, 22	÷	021, 10
y^x	022, 21	STO 9	022, 44 9
STO 6	023, 44 6	x≷y	023, 34
RCL 1	024, 45 1	=	024, 36
RCL 3	025, 45 3	y^x	025, 21
X	026, 20	RCL 4	026, 45 4
RCL 2	027, 45 2	1/x	027, 22
ENTER	028, 36	=	028, 36
X	029, 20	STO 6	029, 44 6
−	030, 30	RCL 9	030, 45 9
RCL 1	031, 45 1	−	031, 30
RCL 3	032, 45 3	RCL 8	032, 45 8
+	033, 40	=	033, 36
RCL 2	034, 45 2	RCL 1	034, 45 1
2	035, 2	X	035, 20
X	036, 20	RCL 3	036, 45 3
−	037, 30	−	037, 30
÷	038, 10	RCL 2	038, 45 2
RCL 4	039, 45 4	g x^2	039, 43 20
÷	040, 10	÷	040, 10
g e^x	041, 43 22	x≷y	041, 34
STO 7	042, 44 7	÷	042, 10
RCL 6	043, 45 6	RCL 4	043, 45 4
1	044, 1	=	044, 36
−	045, 30	g e^x	045, 43 22
RCL 6	046, 45 6	STO 7	046, 44 7

12c platinum / 12C RPN KEYSTROKES	DISPLAY
RCL 4	047, 45 4
y^x	048, 21
1	049, 1
−	050, 30
ENTER	051, 36
X	052, 20
÷	053, 10
RCL 6	054, 45 6
÷	055, 10
RCL 2	056, 45 2
RCL 1	057, 45 1
−	058, 30
X	059, 20
g e^x	060, 43 22
STO 5	061, 44 5
R/S	062, 31
RCL 6	063, 45 6
x≷y	064, 34
y^x	065, 21
RCL 5	066, 45 5
x≷y	067, 34
y^x	068, 21
RCL 7	069, 45 7
X	070, 20
g GTO 062	071,43,33,062
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
RCL 6	047, 45 6
y^x	048, 21
RCL 4	049, 45 4
−	050, 30
1	051, 1
=	052, 36
g x^2	053, 43 20
1	054, 1
−	055, 30
RCL 6	056, 45 6
1/x	057, 22
X	058, 20
RCL 8	059, 45 8
÷	060, 10
x≷y	061, 34
=	062, 36
g e^x	063, 43 22
STO 5	064, 44 5
R/S	065, 31
RCL 6	066, 45 6
y^x	067, 21
x≷y	068, 34
=	069, 36
RCL 5	070, 45 5
y^x	071, 21
x≷y	072, 34
X	073, 20
RCL 7	074, 45 7
=	075, 36
g GTO 065	076,43,33,065
f P/R	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: Unused	R_1: S_1	R_2: S_2
R_3: S_3	R_4: n	R_5: a	R_6: b
R_7: c	R_8-R_0: Unused		

Program Instructions:

1. Key in the program and press f CLEAR REG.
2. Divide the data points to be input into 3 equal consecutive groups. Label them Groups I, II and III for convenience.

3. Key in the first point of group I and press [ENTER]([=])
4. Key in the first point of group II and press [ENTER]([=]).
5. Key in the first point of group III and press [R/S].
6. Repeat steps 3, 4, and 5 for the balance of the data in each group. After executing step 5 the display shows how many sets of data have been entered.
7. To fit the data to a Gompertz curve, press [g][GTO]013[R/S] . The resultant display is the curve constant "*a*". Constants "*b*" and "*c*" may be obtained by pressing [RCL]6 and [RCL]7 respectively. The display may need adjusting to see significant digits.
8. To calculate a projected value, key in the number of the period and press [R/S].
9. Repeat step 8 for each period desired.

Example: The X-presso Company marked a revolutionary new coffee brewing machine in 1990. Sales grew at a steady pace for several years, then began to slow. The sales records for the first 9 years of the product's life were as follows.

Year	Sales($K)	Year	Sales($K)
1	18	6	260
2	41	7	282
3	49	8	322
4	151	9	340
5	188		

What are the projected sales volumes for this product in its 10th and 12th year? What is the maximum yearly sales volume for this product if the present trend continues? What annual sales rate would the curve have predicted for the 5th year of the product's life? (Arrange the data as follows:)

Group I	Group II	Group III
18	151	282
41	188	322
49	260	340

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
[f]CLEAR[REG]	[f]CLEAR[REG]	**0.00**	
18[ENTER]	18[=]	**18.00**	
151[ENTER]	151[=]	**151.00**	
282[R/S]	282[R/S]	**1.00**	
41[ENTER]	41[=]	**41.00**	
188[ENTER]	188[=]	**188.00**	
322[R/S]	322[R/S]	**2.00**	
49[ENTER]	49[=]	**49.00**	
260[ENTER]	260[=]	**260.00**	
340[R/S]	340[R/S]	**3.00**	Total number of entries.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
g GTO 013 R/S	g GTO 013 R/S	**0.004**	a
RCL 6	RCL 6	**0.65**	b
RCL 7	RCL 7	**373.92**	c
10 R/S	10 R/S	**349.09**	Sales in 10th year, (in $K).
12 R/S	12 R/S	**363.36**	Sales in 12th year, (in $K).
100 R/S	100 R/S	**373.92**	Maximum annual sales (after very long product life).
5 R/S	5 R/S	**202.60**	Sales in 5th year (actual sales were $188K).

Forecasting with Exponential Smoothing

A common method for analyzing trends in sales, inventory and securities is the moving average. Exponential smoothing is a version of the weighted moving average which is readily adaptable to programmable calculator forecasting.

Exponential smoothing is often used for short term sales and inventory forecasts. Typical forecast periods are monthly or quarterly. Unlike a moving average, exponential smoothing does not require a great deal of historical data. However , it should not be used with data which has more than a moderate amount of up or down trend.

When using exponential smoothing, a smoothing factor is chosen which affects the sensitivity of the average much the same way as the length of the standard moving average period. The correspondence between the two techniques can be represented by the formula:

$$\alpha = \frac{2}{n+1}$$

where α is the exponential smoothing factor (with values from 0 to 1) and n is the length of the standard moving average. As the equation shows, the longer the moving average period, the smaller the equivalent and the less sensitive the average becomes to fluctuations in current values.

Forecasting with exponential smoothing involves selecting the best smoothing factor based on historical data and then using the factor for updating subsequent data and forecasting. This procedure uses the following HP 12C Platinum program:

12c platinum / 12C RPN KEYSTROKES	DISPLAY	12c platinum ALG KEYSTROKES	DISPLAY
f P/R		f P/R	
f CLEAR PRGM	000,	f CLEAR PRGM	000,
ENTER	001, 36	STO 7	001, 44 7
ENTER	002, 36	−	002, 30
RCL 6	003, 45 6	RCL 6	003, 45 6
−	004, 30	=	004, 36
ENTER	005, 36	R/S	005, 31
X	006, 20	g x^2	006, 43 20
STO + 4	007,44 40 4	STO + 4	007,44 40 4
g LSTx	008, 43 40	RCL 7	008, 45 7
R/S	009, 31	X	009, 20
R↓	010, 33	RCL 0	010, 45 0
R↓	011, 33	=	011, 36
RCL 0	012, 45 0	RCL 2	012, 45 2
X	013, 20	X	013, 20
RCL 2	014, 45 2	RCL 1	014, 45 1
RCL 1	015, 45 1	+	015, 40
X	016, 20	x≷y	016, 34
+	017, 40	−	017, 30
RCL 2	018, 45 2	RCL 2	018, 45 2
CHS	019, 16	X	019, 20
x≷y	020, 34	STO + 2	020,44 40 2
STO 2	021, 44 2	RCL 0	021, 45 0
+	022, 40	=	022, 36
RCL 0	023, 45 0	RCL 3	023, 45 3
X	024, 20	X	024, 20
RCL 1	025, 45 1	RCL 1	025, 45 1
RCL 3	026, 45 3	+	026, 40
X	027, 20	x≷y	027, 34
+	028, 40	÷	028, 10
STO 3	029, 44 3	STO 3	029, 44 3
RCL 1	030, 45 1	RCL 0	030, 45 0
X	031, 20	+	031, 40
RCL 0	032, 45 0	RCL 2	032, 45 2
÷	033, 10	−	033, 30
RCL 2	034, 45 2	STO 6	034, 44 6
+	035, 40	RCL 3	035, 45 3
STO 5	036, 44 5	=	036, 36
RCL 3	037, 45 3	STO 5	037, 44 5

12c platinum / 12C RPN KEYSTROKES	DISPLAY
RCL 0	038, 45 0
÷	039, 10
RCL 2	040, 45 2
+	041, 40
STO 6	042, 44 6
g GTO 000	043,43,33,000
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
RCL 6	038, 45 6
g GTO 000	039,43,33,000
f P/R	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: α	R_1: 1-α	R_2: S_{t-1}
R_3: T_{t-1}	R_4: Σe^2	R_5: D_t	R_6: $\hat{D}_{t+1}$
R_7-R_4: Unused			

Program Instructions:
Selecting the "best" smoothing constant (α):

1. Key in the program and press f CLEAR REG.

RPN Mode:

2. Key in the number 1 and press ENTER.
3. Key in the "trial " and press STO 0 − STO 1.

ALG Mode:

2. Key in the number 1 and press −.
3. Key in the "trial " and press STO 0 = STO 1.

4. Key in the first historical value (X_1) and press STO 2.
5. Key in the second historical value (X_2) and press STO 6 R/S. The result is the error between the forecast value ($\hat{D}_{t+1}$) and the true value (X_{t+1}).
6. Press R/S; the display shows the next forecast ($\hat{D}_{t+2}$).
7. Optional: Press RCL 5 to display the smoothed estimate of current demand.
8. Continue steps 5 and 6 for X_3, X_4, ... X_n until all historical values have been entered. When doing step 5 merely key in the value and press R/S (do not press STO 6).
9. Press RCL 4. This value represents the cumulative forecasting error ($\sum e^2$). Record the value and the following additional values; press RCL 0 (α), RCL 2 (smoothed average S_{t-1}), RCL 3 (trend T_{t-1}) and RCL 6 (forecast $\hat{D}_{t+1}$).
10. Press f CLEAR REG.
11. Repeat steps 2 through 10 until a "best" α is selected based on the lowest cumulative forecasting error (Register 4).

Forecasting:

RPN Mode:

1. Key in the number 1 and press [ENTER].
2. Key in the selected α and press [STO] 0 [−] [STO] 1.

ALG Mode:

1. Key in the number 1 and press [−].
2. Key in the selected α and press [STO] 0 [=] [STO] 1.

3. From the selection routine or from a previous forecast:

   Key in the smoothed average S_{t-1} and press [STO] 2.

   Key in the trend T_{t-1} and press [STO] 3.

   Key in the forecast $\hat{D}_{t+1}$ and press [STO] 6.

4. Key in the current data value and press [R/S]. The output is the error in forecasting the value just entered.
5. Press [R/S]. The displayed value represents the forecast for the next period.
6. Record the following values: [RCL] 0 (α), [RCL] 2 (S_{t-1}), [RCL] 3 (T_{t-1}) and [RCL] 6 ($\hat{D}_{t+1}$) for use as initial values in the next forecast. You may also wish to record [RCL] 5 (D_t).
7. Repeat steps 4, 5, and 6 for the next forecast if available.

Example: Select the best smoothing constant based on sales (in thousands of dollars) of 22, 23, 23, 25, 23, 27, 25. Given the current sales in month 8 of 26, forecast the following month.

Select the smoothing constant (α):

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
[f] CLEAR [REG]	[f] CLEAR [REG]	**0.00**	
1 [ENTER]	1 [−]	**1.00**	
.5 [STO] 0 [−]	.5 [STO] 0 [=]	**0.50**	
[STO] 1	[STO] 1	**0.50**	
22 [STO] 2	22 [STO] 2	**22.00**	
23 [STO] 6 [R/S]	23 [STO] 6 [R/S]	**0.00**	
[R/S]	[R/S]	**23.00**	
23 [R/S] [R/S]	23 [R/S] [R/S]	**23.25**	
25 [R/S] [R/S]	25 [R/S] [R/S]	**25.25**	
23 [R/S] [R/S]	23 [R/S] [R/S]	**23.69**	
27 [R/S] [R/S]	27 [R/S] [R/S]	**27.13**	
25 [R/S] [R/S]	25 [R/S] [R/S]	**25.95**	
[RCL] 4	[RCL] 4	**23.61**	Cumulative error ($\sum e^2$).
[RCL] 0	[RCL] 0	**0.50**	Smoothing constant (α).

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
RCL 2	RCL 2	**25.11**	Smoothing average (S_{t-1}).
RCL 3	RCL 3	**0.42**	Trend (T_{t-1}).
RCL 6	RCL 6	**25.95**	Last forecast ($\hat{D}_{t+1}$).

The procedure is repeated for several α's.

Smoothing Constant (α)	.5	.1	.25	.2
Cumulative Error (Σe^2)	23.61	25.14	17.01	18.03
For the selected α = .25	S_{t+1} =	24.28		
	T_{t-1} =	0.34		
	$\hat{D}_{t+1}$=	25.64		

Forecasting:

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
f CLEAR REG	f CLEAR REG	**0.00**	
1 ENTER	1 −	**1.00**	
.25 STO 0 −	.25 STO 0 =	**0.75**	
STO 1	STO 1	**0.75**	
24.28 STO 2	24.28 STO 2	**24.28**	
.34 STO 3	.34 STO 3	**0.34**	
25.64 STO 6	25.64 STO 6	**25.64**	
26 R/S	26 R/S	**0.36**	
R/S	R/S	**26.16**	Forecast for month 9, ($\hat{D}_{t+1}$).
RCL 5	RCL 5	**25.80**	Expected usage for current (month 8) period, (Smoothed D_t).
RCL 0	RCL 0	**0.25**	α
RCL 2	RCL 2	**24.71**	Record for initial values when month 9 actual figures become available.
RCL 3	RCL 3	**0.36**	
RCL 6	RCL 6	**26.16**	

Note: At least 4 periods of current data should be entered before forecasting is attempted.

Pricing Calculations

Markup and Margin Calculations

Sales work often involves calculating the various relations between markup, margin, selling price and costs. Markup is defined as the difference between selling price and cost, divided by the cost. Margin is defined as the difference between selling price and cost, divided by selling price. In other words, markup is based on cost and margin is based on selling price.

The following keystroke sequences are given to readily make these calculations on the HP 12C Platinum.

CALCULATE	GIVEN	RPN KEYSTROKES	ALG KEYSTROKES
Selling Price	Cost & Markup	Key in cost, [ENTER], key in markup (in %), [%] [+].	Key in cost, [+], key in markup (in %), [%] [=].
Selling Price	Cost & Margin	Key in cost, [ENTER] 1 [ENTER] key in margin (in %), [%] [−] [÷].	Key in cost, [1/x] [−], key in margin (in %), [%] [=] [1/x].
Cost	Selling Price & Markup	Key in selling price, [ENTER] 1 [ENTER], key in markup (in %), [%] [+] [÷].	Key in selling price, [1/x] [+], key in markup (in %), [%] [=] [1/x].
Cost	Selling Price & Margin	Key in selling price, [ENTER], key in margin (in %), [%] [−].	Key in selling price, [−], key in margin (in %), [%] [=].
Markup	Cost and Selling Price	Key in cost, [ENTER], key in selling price, [Δ%].	Key in cost, [=], key in selling price, [Δ%].
Markup	Margin	Key in margin, [ENTER] [ENTER] 1 [x≷y] [%] [−] [÷].	Key in margin, [−] [1/x] [x≷y] [%] [=] [1/x].
Margin	Selling Price & Cost	Key in selling price, [ENTER], key in cost, [Δ%] [CHS].	Key in selling price, [=], key in cost, [Δ%] [CHS].
Margin	Markup	Key in markup, [ENTER] [ENTER] 1 [x≷y] [%] [+] [÷].	Key in markup, [+] [1/x] [x≷y] [%] [=] [1/x].

Example 1: If the cost of an item is $160 and the margin is 20%, what is the selling price? What is the markup?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
160 ENTER 1 ENTER 20	160 1/x − 20	**20.**	Margin(%).
% − ÷	% = 1/x	**200.00**	Selling price.
20 ENTER ENTER	20 −	**20.00**	Margin(%)
1 x≷y % − ÷	1/x x≷y % = 1/x	**25.00**	Markup (%).

Example 2: If an item sells for $21.00 and has a markup of 50%, what is its cost? What is the margin?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
21 ENTER 1 ENTER 50	21 1/x + 50	**50.00**	Markup (%).
% + ÷	% = 1/x	**14.00**	Cost.
50 ENTER ENTER	50 +	**50.00**	Markup(%)
1 x≷y % + ÷	1/x x≷y % = 1/x	**33.33**	Margin (%).

The following HP 12C Platinum program may be helpful for repetitive calculations of selling price and costs as well as conversions between markup and margin.

12c platinum / 12C RPN KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
ENTER	001, 36
g GTO 004	002,43,33,004
CHS	003, 16
1	004, 1
x≷y	005, 34
%	006, 25
+	007, 40
÷	008, 10
R/S	009, 31
g LSTx	010, 43 40
X	011, 20
g LSTx	012, 43 40
X	013, 20
g GTO 000	014,43,33,000
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
=	001, 36
g GTO 004	002,43,33,004
CHS	003, 16
x≷y	004, 34
+	005, 40
x≷y	006, 34
%	007, 25
=	008, 36
g LSTx	009, 43 40
g x^2	010, 43 20
÷	011, 10
x≷y	012, 34
=	013, 36
R/S	014, 31
g LSTx	015, 43 40
g GTO 000	016,43,33,000
f P/R	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0-R_8: Unused		

Program Instructions:

1. Key in the program.
2. To calculate selling price, given the markup, key in the cost, press ENTER(=), key in the markup and press g GTO 000 R/S R/S.
3. To calculate cost, given the markup, key in the selling price, press ENTER(=), key in the markup and press g GTO 000 R/S.
4. To calculate selling price, given the margin, key in the cost, press ENTER(=), key in the margin and press g GTO 003 R/S.
5. To calculate cost, given the margin, key in the selling price, press ENTER(=), key in the margin and press g GTO 003 R/S R/S.

RPN Mode:

6. To calculate markup from the margin, key in the margin and press ENTER g GTO 003 R/S.
7. To calculate margin from the markup, key in the markup and press ENTER g GTO 000 R/S.

ALG Mode:

6. To calculate markup from the margin, key in the margin and press =, re-key margin, g GTO 003 R/S.
7. To calculate margin from the markup, key in the markup and press =, re-key markup, g GTO 000 R/S.

Example: Find the cost of an item selling for $38.00 with a margin of 30%. What is the markup on the item? If the markup is raised to 50%, what will the selling price be?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
38 ENTER	38 =	**38.00**	Selling price.
30 g GTO 003	30 g GTO 003	**30.00**	Margin (%).
R/S R/S	R/S R/S	**26.60**	Cost.
30 ENTER	30 =	**30.00**	Margin (%).
g GTO 003 R/S	30 g GTO 003 R/S	**42.86**	Markup (%).
26.6 ENTER	26.6 =	**26.60**	Cost.
50 g GTO 000	50 g GTO 000	**50.00**	New markup.
R/S R/S	R/S R/S	**39.90**	New selling price.

Calculations of List and Net prices With Discounts

It is often useful to be able to quickly calculate a list or net price when the other price and a series of discount rates are known. Alternatively, if the list and net price and several discounts are known it may be desirable to calculate a missing discount. The following series of keystrokes may be used:

RPN Mode:

1. Key in 1, press [ENTER] [ENTER] [STO] 1.
2. Key in the first discount (as a percentage) and press [%] [−] [STO] [X] 1 [R↓].
3. Repeat step 2 for each of the remaining known discount rates.
4. To calculate the list price, key in the net price and press [RCL] 1 [÷].
5. To calculate the net price, key in the list price and press [RCL] 1 [X].
6. To calculate an unknown discount rate, immediately after step 3 (display should show 1.00), key in the net price, press [ENTER] and key in the list price.
7. Press [RCL] 1 [X] [÷] [−] 100 [X].

ALG Mode:

1. Key in 1, press [STO] 1.
2. Key in 1 [−], key in the first discount (as a percentage) and press [%] [=] [STO] [X] 1.
3. Repeat step 2 for each of the remaining known discount rates.
4. To calculate the list price, key in the net price and press [÷] [RCL] 1 [=].
5. To calculate the net price, key in the list price and press [X] [RCL] 1 [=].
6. To calculate an unknown discount rate, immediately after doing step 3 , key in the list price, press [X] [RCL] 1 [=], then key in the net price.
7. Press [Δ%] [CHS].

Example: The list price of an item is $3.28 and the net price is $1.45. Two of the discount rates are 48% and 5%. What is the third discount rate?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
1 [ENTER] [ENTER]	1		
[STO] 1	[STO] 1	**1.00**	
48 [%] [−] [STO] [X] 1	1 [−] 48 [%] [=] [STO] [X] 1	**0.52**	
[R↓] 5 [%] [−] [STO] [X] 1	1 [−] 5 [%] [=] [STO] [X] 1	**0.95**	
[R↓] 1.45 [ENTER]			
3.28 [RCL] 1	3.28 [X] [RCL] 1	**0.49**	
[X] [÷] [−] 100 [X]	[=] 1.45 [Δ%] [CHS]	**10.51**	3rd discount rate (%).

The following program for the HP 12C Platinum will be helpful in performing the calculations:

12c platinum / 12C RPN KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
1	001, 1
x≷y	002, 34
%	003, 25
−	004, 30
STO X 1	005,44 20 1
g GTO 000	006,43,33,000
RCL 1	007, 45 1
X	008, 20
÷	009, 10
1	010, 1
x≷y	011, 34
−	012, 30
EEX	013, 26
2	014, 2
X	015, 20
g GTO 000	016,43,33,000
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
1	001, 1
−	002, 30
x≷y	003, 34
%	004, 25
=	005, 36
STO X 1	006,44 20 1
g GTO 000	007,43,33,000
RCL 1	008, 45 1
X	009, 20
x≷y	010, 34
=	011, 36
x≷y	012, 34
Δ%	013, 24
CHS	014, 16
g GTO 000	015,43,33,000
f P/R	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: Unused	R_1: $D'_1 x D'_2 ...$	R_2-R_7: Unused

Program Instructions:

1. Key in the program.
2. Key in 1 and press STO 1.
3. Key in the first discount rate (as a percentage) and press R/S.
4. Repeat step 2 for each of the remaining discount rates.

RPN Mode:

5. To calculate the list price, key in the net price and press RCL 1 ÷.
6. To calculate the net price, key in the list price and press RCL 1 X.
7. To calculate the unknown discount rate, key in the net price, press ENTER, key in the list price and press g GTO 007 R/S.

ALG Mode:

5. To calculate the list price, key in the net price and press ÷ RCL 1 =.
6. To calculate the net price, key in the list price and press X RCL 1 =.
7. To calculate the unknown discount rate, key in the net price, press =, key in the list price and press g GTO 008 R/S.

Example: Calculate the unknown discount rate for the previous example. If the list price is now raised to $3.75 what is the new net price?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
1 STO 1	1 STO 1	**1.00**	
48 R/S	48 R/S	**0.52**	
5 R/S	5 R/S	**0.95**	
1.45 ENTER	1.45 =		
3.28 g GTO 007 R/S	3.28 g GTO 008 R/S	**10.51**	3rd discount rate (%).
R/S	R/S	**0.89**	Include 3rd discount rate in calculation.
3.75 RCL 1 X	3.75 X RCL 1 =	**1.66**	New net price.

Statistics

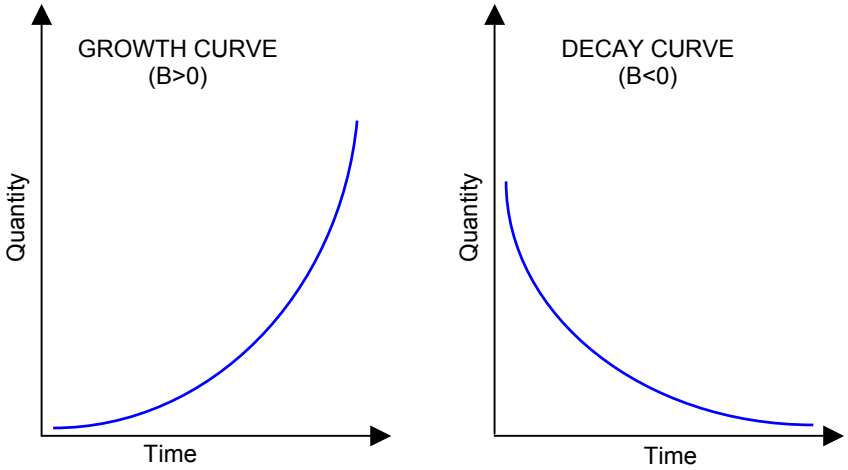
Curve Fitting

Exponential Curve Fit

Using the [LN] function of the HP 12C Platinum, a least squares exponential curve fit may be easily calculated according to the equation $y=Ae^{Bx}$. The exponential curve fitting technique is often used to determine the growth rate of a variable such as a stock's value over time, when it is suspected that the performance is non-linear. The value for *B* is the decimal value of the continuous growth rate. For instance, assume after keying in several end-of-month price quotes for a particular stock it is determined that the value of *B* is 0.10. This means that over the measured growth period the stock has experienced a 10% continuous growth rate.

If $B>0$, you will have a growth curve. If $B<0$, you will have a decay curve.

Examples of these are given below.



The procedure is as follows:

1. Press [f] CLEAR [REG] .
2. For each input pair of values, key in the y-value and press [g] [LN], key in the corresponding x-value and press [Σ+] .
3. After all data pairs are input, press [g] [ŷ,r] [x≷y] to obtain the correlation coefficient (between ln y and x).

4. Press 1 [g] [ŷ,r] [g] [e^x] 0 [g] [ŷ,r] [g] [e^x] to obtain A in the equation above.

RPN Mode:

5. Press [x≷y] [R↓] [÷] [g] [LN] to obtain B.
6. Press [g] [e^x] 1 [−] to obtain the effective growth rate (as a decimal).

ALG Mode:

5. Press [÷] [x≷y] [R↓] [=] [g] [LN] to obtain B.
6. Press [g] [e^x] [−] 1 [=] to obtain the effective growth rate (as a decimal).

7. To make a y-estimate, key in the x-value and press [g] [ŷ,r] [g] [e^x].

Example 1: A stock's price in history is listed below. What effective growth rate does this represent? If the stock continues this growth rate, what is the price projected to be at the end of 2004 (year 7)?

End of Year	Price
1998(1)	45
1999(2)	51.5
2000(3)	53.75
2001(4)	80
2002(5)	122.5
2003(6)	210
2004(7)	?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
[f] CLEAR [REG] 45 [g] [LN] 1 [Σ+]	[f] CLEAR [REG] 45 [g] [LN] 1 [Σ+]	**1.00**	First data pair input.
51.5 [g] [LN] 2 [Σ+]	51.5 [g] [LN] 2 [Σ+]	**2.00**	Second data pair input.
53.75 [g] [LN] 3 [Σ+]	53.75 [g] [LN] 3 [Σ+]	**3.00**	Third data pair input.
80 [g] [LN] 4 [Σ+]	80 [g] [LN] 4 [Σ+]	**4.00**	Fourth data pair input.
122.5 [g] [LN] 5 [Σ+]	122.5 [g] [LN] 5 [Σ+]	**5.00**	Fifth data pair input.
210 [g] [LN] 6 [Σ+]	210 [g] [LN] 6 [Σ+]	**6.00**	Sixth data pair input.
[g] [ŷ,r] [x≷y]	[g] [ŷ,r] [x≷y]	**0.95**	Correlation coefficient (between ln *y* and *x*).
1 [g] [ŷ,r] [g] [e^x] 0 [g] [ŷ,r] [g] [e^x]	1 [g] [ŷ,r] [g] [e^x] 0 [g] [ŷ,r] [g] [e^x]	**27.34**	A
[x≷y] [R↓] [÷] [g] [LN]	[÷] [x≷y] [R↓] [=] [g] [LN]	**0.31**	B
[g] [e^x] 1 [−]	[g] [e^x] [−] 1 [=]	**0.36**	Effective growth rate.
7 [g] [ŷ,r] [g] [e^x]	7 [g] [ŷ,r] [g] [e^x]	**232.35**	Projected price at end of year 7 (2004).

For repeated use of this routine, the following HP 12C Platinum program will be useful.

12c platinum / 12C RPN KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
x≷y	001, 34
g LN	002, 43 23
x≷y	003, 34
Σ+	004, 49
g GTO 000	005,43,33,000
g ŷ,r	006, 43 2
x≷y	007, 34
R/S	008, 31
1	009, 1
g ŷ,r	010, 43 2
g e^x	011, 43 22
0	012, 0
g ŷ,r	013, 43 2
g e^x	014, 43 22
R/S	015, 31
x≷y	016, 34
R↓	017, 33
÷	018, 10
g LN	019, 43 23
R/S	020, 31
g e^x	021, 43 22
1	022, 1
−	023, 30
R/S	024, 31
g ŷ,r	025, 43 2
g e^x	026, 43 22
g GTO 000	027,43,33,000
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
x≷y	001, 34
g LN	002, 43 23
x≷y	003, 34
Σ+	004, 49
g GTO 000	005,43,33,000
g ŷ,r	006, 43 2
x≷y	007, 34
R/S	008, 31
0	009, 0
g ŷ,r	010, 43 2
g e^x	011, 43 22
R/S	012, 31
1	013, 1
g ŷ,r	014, 43 2
g e^x	015, 43 22
x≷y	016, 34
R↓	017, 33
÷	018, 10
x≷y	019, 34
=	020, 36
g LN	021, 43 23
R/S	022, 31
g e^x	023, 43 22
−	024, 30
1	025, 1
=	026, 36
R/S	027, 31
g ŷ,r	028, 43 2
g e^x	029, 43 22
g GTO 000	030,43,33,000
f P/R	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: Unused	R_1: n	R_2: Σx
R_3: Σx^2	R_4: Σy	R_5: Σy^2	R_6: Σxy
R_7-$R_{.6}$: Unused			

Program Instructions:

1. Key in the program and press [f] CLEAR [REG].
2. For each input pair of values, key in the y-value and press [ENTER] ([=]), key in the corresponding *x*- value and press [R/S].
3. After all data pairs are input, press [g] [GTO] 006 [R/S] to obtain the correlation coefficient (between ln *y* and *x*).
4. Press [R/S] to obtain *A*.
5. Press [R/S] to obtain *B*.
6. Press [R/S] to obtain the effective growth rate as a decimal.
7. **RPN:** To make a y-estimate, key in the x-value and press [R/S]. For subsequent estimates, key in the x-value and press [g] [GTO] 025 [R/S].

7. **ALG:** To make a y-estimate, key in the x-value and press [R/S]. For subsequent estimates, key in the x-value and press [g] [GTO] 028 [R/S].

8. For a different set of data, press [f] CLEAR [REG] and go to step 2.

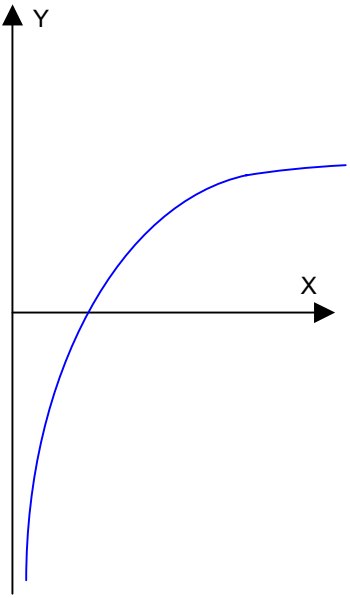
Example 2: Repeat example 1 using the program.

12c platinum /12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
[f] CLEAR [REG]	[f] CLEAR [REG]		
45 [ENTER] 1 [R/S]	45 [=] 1 [R/S]	**1.00**	First data pair input.
51.5 [ENTER] 2 [R/S]	51.5 [=] 2 [R/S]	**2.00**	Second data pair input.
53.75 [ENTER] 3 [R/S]	53.75 [=] 3 [R/S]	**3.00**	Third data pair input.
80 [ENTER] 4 [R/S]	80 [=] 4 [R/S]	**4.00**	Fourth data pair input.
122.5 [ENTER] 5 [R/S]	122.5 [=] 5 [R/S]	**5.00**	Fifth data pair input.
210 [ENTER] 6 [R/S]	210 [=] 6 [R/S]	**6.00**	Sixth data pair input.
[g] [GTO] 006 [R/S]	[g] [GTO] 006 [R/S]	**0.95**	Correlation coefficient (between ln *y* and *x*).
[R/S]	[R/S]	**27.34**	A
[R/S]	[R/S]	**0.31**	B
[R/S]	[R/S]	**0.36**	Effective growth rate.
7 [R/S]	7 [R/S]	**232.35**	Projected price at the end of year 7 (2004).

Logarithmic Curve Fit

If your data does not fit a line or an exponential curve, try the following logarithmic curve fit. This is calculated according to the equation $y = A + B\ (\ln x)$, and all x values must be positive.

A typical logarithmic curve is shown below.



The procedure is as follows:

1. Press [f] CLEAR [REG].
2. Key in the first *y*-value and press [ENTER] ([=]). Key in the first *x*-value and press [g] [LN] [Σ+]. Repeat this step for each data pair.
3. After all data pairs are input, press [g] [ŷ,r] [x≷y] to obtain the correlation coefficient (between *y* and ln *x*).
4. Press 1 [g] [ŷ,r] 0 [g] [ŷ,r] to obtain *A* in the equation above.
5. **RPN:** Press [x≷y] [R↓] [−] to obtain *B*.
5. **ALG:** Press [−] [x≷y] [R↓] [=] to obtain *B*.
6. To make a *y*-estimate, key in the *x*-value and press [g] [LN] [g] [ŷ,r].

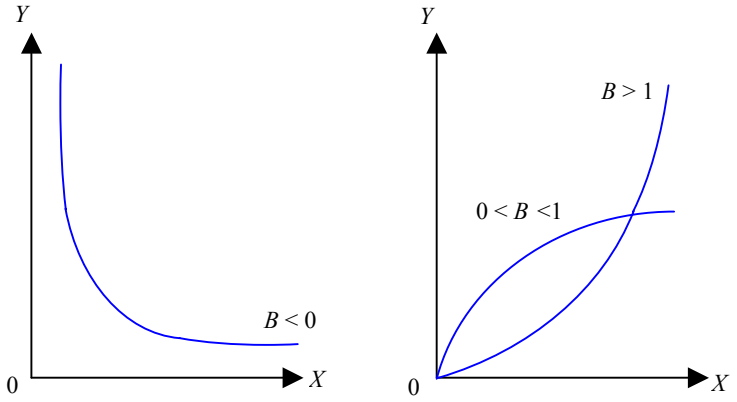
Example 1: A manufacturer observes declining sales of a soon-to-be obsolete product, of which there were originally 10,000 units in inventory. The cumulative sales figures over a number of months, given below, may be approximated by a logarithmic curve of the form $y = A + B\ (\ln x)$, where y represents cumulative sales in units and x the number of months since the beginning. How many units will be sold by the end of the eighth month?

Month	1	2	3	4	5	6
Cumulative Sales (units)	1431	3506	5177	6658	7810	8592

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
f CLEAR REG 1431 ENTER 1 g LN Σ+	f CLEAR REG 1431 = 1 g LN Σ+	**1.00**	First pair data input.
3506 ENTER 2 g LN Σ+	3506 = 2 g LN Σ+	**2.00**	Second pair data input.
5177 ENTER 3 g LN Σ+	5177 = 3 g LN Σ+	**3.00**	Third pair data input.
6658 ENTER 4 g LN Σ+	6658 = 4 g LN Σ+	**4.00**	Fourth pair data input.
7810 ENTER 5 g LN Σ+	7810 = 5 g LN Σ+	**5.00**	Fifth pair data input.
8592 ENTER 6 g LN Σ+	8592 = 6 g LN Σ+	**6.00**	Sixth pair data input.
g ŷ,r x≷y	g ŷ,r x≷y	**0.99**	Correlation coefficient (between y and ln x).
1 g ŷ,r 0 g ŷ,r	1 g ŷ,r 0 g ŷ,r	**1,066.15**	Value of A.
x≷y R↓ −	− x≷y R↓ =	**4,069.93**	Value of B.
8 g LN g ŷ,r	8 g LN g ŷ,r	**9,529.34**	Total units sold by end of eighth month.

Power Curve Fit

Another method of analysis is the power curve or geometric curve. The equation of the power curve is $y = Ax^B$, and the values for A and B are computed by calculations similar to linear regression. Some examples of power curves are shown below.



The following keystrokes fit a power curve according to the equation $\ln y = \ln A + B(\ln x)$:

1. Press [f] CLEAR [REG].
2. Key in the first y-value and press [g] [LN]. Key in the first x-value and press [g] [LN] [Σ+]. Repeat this step for all data pairs.
3. Press [g] [ŷ,r] [x≷y], to obtain the correlation coefficient (between $\ln y$ and $\ln x$).
4. Press 0 [g] [ŷ,r] [g] [e^x] to obtain A in the above equation.
5. **RPN:** Press 1 [g] [ŷ,r] 0 [g] [ŷ,r] [x≷y] [R↓] [−] to obtain B.
5. **ALG:** Press 1 [g] [ŷ,r] 0 [g] [ŷ,r] [−] [x≷y] [R↓] [=] to obtain B.
6. To make a y-estimate, key in the x-value and press [g] [LN] [g] [ŷ,r] [g] [e^x].

Example: If Galileo had wished to investigate quantitatively the relationship between the time (t) for a falling object to hit the ground and the height (h) it has fallen, he might have released a rock from various levels of the Tower of Pisa (which was leaning even then) and timed its descent by counting his pulse. The following data are measurements Galileo might have made.

t (pulses)	2	2.5	3.5	4	4.5
h (feet)	30	50	90	130	150

Find the power curve formulas that best expresses h as a function of t ($h = At^B$).

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
f CLEAR REG 30 g LN 2 g LN Σ+	f CLEAR REG 30 g LN 2 g LN Σ+	**1.00**	First pair data input.
50 g LN 2.5 g LN Σ+	50 g LN 2.5 g LN Σ+	**2.00**	Second pair data input.
90 g LN 3.5 g LN Σ+	90 g LN 3.5 g LN Σ+	**3.00**	Third pair data input.
130 g LN 4 g LN Σ+	130 g LN 4 g LN Σ+	**4.00**	Fourth pair data input.
150 g LN 4.5 g LN Σ+	150 g LN 4.5 g LN Σ+	**5.00**	Fifth pair data input.
g ŷ,r x≷y	g ŷ,r x≷y	**1.00**	Correlation coefficient (between ln y and ln x).
0 g ŷ,r g e^x	0 g ŷ,r g e^x	**7.72**	Value of A.
1 g ŷ,r 0 g ŷ,r x≷y R↓ −	1 g ŷ,r 0 g ŷ,r − x≷y R↓ =	**1.99**	Value of B.

The formula that best expresses h as a function of t is

$$h = 7.72t^{1.99}$$

We know, as Galileo did not, that in fact h is proportional to t^2.

Standard Error of the Mean

The standard error of the mean is a measure of how reliable the mean of a sample (X) is as an estimator of the mean of the population from which the sample was drawn.

To calculate the standard error of the mean:

1. Press f CLEAR REG.
2. If you are summing one set of numbers, key in the first number and press Σ+. Continue until you have entered all of the values.
3. If you are summing two sets of numbers, key in the y-value and press ENTER (=), key in the x-value and press Σ+. Continue until you have entered all of the values.
4. Press g x̄ to obtain the mean of the x-values.

RPN Mode:

5. Press g s RCL 1 g √x ÷ to obtain the standard error of the mean of the x-values.
6. Alternatively, press g s x≷y RCL 1 g √x ÷ to obtain the standard error for the mean of the y-values.

ALG Mode:

5. Press g s ÷ RCL 1 g √x = to obtain the standard error of the mean of the *x*-values.
6. Alternatively, press g s x≷y ÷ RCL 1 g √x = to obtain the standard error for the mean of the *y*-values.

Example: A sample of 6 one-bedroom apartment rentals reveals that one rents for $190 per month unfurnished, one rents for $200 per month, two rent for $205 per month, one rents for $216 per month, and one rents for $220 per month. What are the mean monthly rental and the standard deviation? What is the standard error of the mean?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
f CLEAR REG 190 Σ+ 200 Σ+ 205 Σ+ 205 Σ+ 216 Σ+ 220 Σ+	f CLEAR REG 190 Σ+ 200 Σ+ 205 Σ+ 205 Σ+ 216 Σ+ 220 Σ+	**6.00**	Total number of inputs.
g x̄	g x̄	**206.00**	Average monthly rent.
g s	g s	**10.86**	Standard deviation.
RCL 1 g √x ÷	÷ RCL 1 g √x =	**4.43**	Standard error of the mean.

Mean, Standard Deviation, Standard Error for Grouped Data

Grouped data are presented in frequency distributions to save time and effort in writing down (or entering) each observation individually. Given a set of data points

$$x_1, x_2, \ldots, x_n$$

with respective frequencies

$$f_1, f_2, \ldots, f_n$$

this procedure computes the mean, standard deviation, and standard error of the mean.

1. Press f CLEAR REG.

RPN Mode:

2. Key in the first value and press ENTER ENTER.
3. Key in the respective frequency and press STO + 0 X Σ+. The display shows the number of data points entered.

ALG Mode:

2. Key in the first value and press =.
3. Key in the respective frequency and press STO + 0 X x≷y = g LSTx x≷y Σ+. The display shows the number of data points entered.

4. Repeat steps 2 and 3 for each data point.

5. To calculate the mean (average) press RCL 0 STO 1 RCL 6 STO 3 g $\bar{x}$.
6. Press g s to find the standard deviation.
7. **RPN:** Press RCL 0 g $\sqrt{x}$ ÷ to find the standard error of the mean.
7. **ALG:** Press ÷ RCL 0 g $\sqrt{x}$ = to find the standard error of the mean.

Example 1: A survey of 266 one-bedroom apartment rentals reveals that 54 rent for $190 a month unfurnished, 32 rent for $195 per month, 88 rent for $200 per month, and 92 rent for 206 per month. What are the average monthly rental, the standard deviation, and the standard error of the mean?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
f CLEAR REG 190 ENTER ENTER 54 STO + 0 X Σ+	f CLEAR REG 190 = 54 STO + 0 X x≷y = g LSTx x≷y Σ+	**1.00**	First data pair entered.
195 ENTER ENTER 32 STO + 0 X Σ+	195 = 32 STO + 0 X x≷y = g LSTx x≷y Σ+	**2.00**	Second data pair entered.
200 ENTER ENTER 88 STO + 0 X Σ+	200 = 88 STO + 0 X x≷y = g LSTx x≷y Σ+	**3.00**	Third data pair entered.
206 ENTER ENTER 92 STO + 0 X Σ+	206 = 92 STO + 0 X x≷y = g LSTx x≷y Σ+	**4.00**	Fourth data pair entered.
RCL 0 STO 1 RCL 6 STO 3 g $\bar{x}$	RCL 0 STO 1 RCL 6 STO 3 g $\bar{x}$	**199.44**	Average monthly rent.
g s	g s	**5.97**	Standard deviation.
RCL 0 g $\sqrt{x}$ ÷	÷ RCL 0 g $\sqrt{x}$ =	**0.37**	Standard error of the mean.

Use the following HP 12C Platinum program for the previous example:

12c platinum / 12C RPN KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
STO + 0	001,44 40 0
X	002, 20
Σ+	003, 49
g GTO 000	004,43,33,000
RCL 0	005, 45 0
STO 1	006, 44 1
RCL 6	007, 45 6
STO 3	008, 44 3
g $\bar{x}$	009, 43 0
R/S	010, 31

12c platinum ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
STO + 0	001,44 40 0
X	002, 20
x≷y	003, 34
=	004, 36
g LSTx	005, 43 40
x≷y	006, 34
Σ+	007, 49
g GTO 000	008,43,33,000
RCL 0	009, 45 0
STO 1	010, 44 1

12c platinum / 12C RPN KEYSTROKES	DISPLAY
g s	011, 43 48
R/S	012, 31
RCL 0	013, 45 0
g √x	014, 43 21
÷	015, 10
g GTO 000	016,43,33,000
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
RCL 6	011, 45 6
STO 3	012, 44 3
g x̄	013, 43 0
R/S	014, 31
g s	015, 43 48
R/S	016, 31
÷	017, 10
RCL 0	018, 45 0
g √x	019, 43 21
=	020, 36
g GTO 000	021,43,33,000
f P/R	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: Σf_i	R_1: Σf_i	R_2: $\Sigma f_i x_i$
R_3: $\Sigma f_i x_i^2$	R_4: Σx_i	R_5: Σx_i^2	R_6: $\Sigma f_i x_i^2$
R_7-$R_{.7}$: Unused			

Program Instructions:

1. Key in the program.
2. Press f CLEAR REG.
3. **RPN:** Key in the first value and press ENTER ENTER.
3. **ALG:** Key in the first value and press =.
4. Key in the respective frequency and press R/S. The display shows the number of data points entered.
5. Repeat steps 3 and 4 for each data point.
6. **RPN:** To calculate the mean, press g GTO 005 R/S.
6. **ALG:** To calculate the mean, press g GTO 009 R/S.
7. Press R/S to find the standard deviation.
8. Press R/S to find the standard error of the mean.
9. For a new case, go to step 2.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
f CLEAR REG 190 ENTER ENTER 54 R/S	f CLEAR REG 190 = 54 R/S	**1.00**	First data pair.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
195 ENTER ENTER 32 R/S	195 = 32 R/S	**2.00**	Second data pair.
200 ENTER ENTER 88 R/S	200 = 88 R/S	**3.00**	Third data pair.
206 ENTER ENTER 92 R/S	206 = 92 R/S	**4.00**	Total number of data sets.
g GTO 005 R/S	g GTO 009 R/S	**199.44**	Average monthly rent (mean).
R/S	R/S	**5.97**	Standard deviation.
R/S	R/S	**0.37**	Standard error of the mean.

Chi-Square Statistics

The chi-square statistic is a measure of the goodness of fit between two sets of frequencies. It is used to test whether a set of observed frequencies differs from a set of expected frequencies sufficiently to reject the hypothesis under which the expected frequencies were obtained.

In other words, you are testing whether discrepancies between the observed frequencies (O_i) and the expected frequencies (E_i) are significant, or whether they may reasonable be attributed to chance. The formula generally used is:

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

If there is a close agreement between the observed and expected frequencies, χ^2 will be small. If the agreement is poor, χ^2 will be large.

The following keystrokes calculate the χ^2 statistic:

1. Press f CLEAR REG .

RPN Mode:

2. Key in the first O_i value and press ENTER .
3. Key in the first E_i value and press STO 0 − ENTER X RCL 0 ÷ + .

ALG Mode:

2. Key in the first O_i value and press − .
3. Key in the first E_i value and press STO 0 = g x^2 ÷ RCL 0 + x≷y = .

4. Repeat steps 2 and 3 for all data pairs. The χ^2 value is displayed.

Example 1: A suspect die from a Las Vegas casino is brought to an independent testing firm to determine its bias, if any. The die is tossed 120 times and the following results obtained.

Number	1	2	3	4	5	6
Observed Frequency	25	17	15	23	24	16

The expected frequency = 120 throws / 6 sides, or $E = 20$ for each number, 1 thru 6. (Since E is a constant in this example, there is no need to store it in R_0 each time.)

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
f CLEAR REG	f CLEAR REG		
25 ENTER	25 −		
20 STO 0 − ENTER	20 STO 0 = g x^2		
X RCL 0 ÷ +	÷ RCL 0 + x≷y =	**1.25**	
17 ENTER 20 −	17 − 20 = g x^2		
ENTER X RCL 0 ÷ +	÷ RCL 0 + x≷y =	**1.70**	
15 ENTER 20 −	15 − 20 = g x^2		
ENTER X RCL 0 ÷ +	÷ RCL 0 + x≷y =	**2.95**	
23 ENTER 20 −	23 − 20 = g x^2		
ENTER X RCL 0 ÷ +	÷ RCL 0 + x≷y =	**3.40**	
24 ENTER 20 −	24 − 20 = g x^2		
ENTER X RCL 0 ÷ +	÷ RCL 0 + x≷y =	**4.20**	
16 ENTER 20 −	16 − 20 = g x^2		
ENTER X RCL 0 ÷ +	÷ RCL 0 + x≷y =	**5.00**	χ^2

The number of degrees of freedom is (n-1). Since $n = 6$, the degrees of freedom = 5.

Consulting statistical tables, you look up χ^2 to a 0.05 significance level with 5 degrees of freedom, and see that $\chi^2_{0.05,5} = 11.07$. Since $\chi^2 = 5$ is within 11.07, we may conclude that to a 0.05 significance level (probability = .95), the die is fair.

Try the following HP 12C Platinum program with the same example.

12c platinum / 12C RPN KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
STO 0	001, 44 0
−	002, 30
ENTER	003, 36
X	004, 20
RCL 0	005, 45 0
÷	006, 10
+	007, 40
g GTO 000	008,43,33,000
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
x≷y	001, 34
Δ%	002, 24
%	003, 25
g x^2	004, 43 20
X	005, 20
x≷y	006, 34
+	007, 40
x≷y	008, 34
=	009, 36
g GTO 000	010,43,33,000
f P/R	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: E_i	R_1-R_9: Unused	

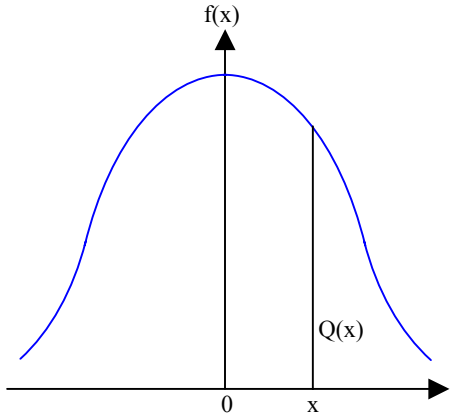
Program Instructions:

1. Key in the program.
2. Press f CLEAR REG.
3. Key in the first O_i value and press ENTER (=).
4. Key in the first E_i value and press R/S.
5. Repeat steps 3 and 4 for all data pairs. The χ^2 value is displayed.
6. For a new case, go to step 2.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
f CLEAR REG	f CLEAR REG		
25 ENTER	25 =		
20 R/S	20 R/S	1.25	
17 ENTER	17 =		
20 R/S	20 R/S	1.70	
15 ENTER	15 =		
20 R/S	20 R/S	2.95	
23 ENTER	23 =		
20 R/S	20 R/S	3.40	
24 ENTER	24 =		
20 R/S	20 R/S	4.20	
16 ENTER	16 =		
20 R/S	20 R/S	5.00	χ^2

Normal Distribution

The normal (or Gaussian) distribution is an important tool in statistics and business analysis. The following HP 12C Platinum program gives an *approximation* to the upper tail area Q under a standardized normal distribution curve, given x. The upper tail area signifies the probability of occurrence of all values $\geq x$.



$$Q(x) \cong \frac{1}{2} EXP\left[-\frac{(83x+351)+562}{703/x+165}\right]$$

Relative error less than 0.042% over the range $0 < x < 5.5$

Reference:

Stephen E. Derenzo, "Approximations for Hand Calculators Using Small Integer Coefficients," *Mathematics of Computation*, Vol. 31, No. 137, page 2014,225; Jan 1977.

12c platinum / 12C RPN KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
STO 0	001, 44 0
8	002, 8
3	003, 3
X	004, 20
3	005, 3
5	006, 5
1	007, 1

12c platinum ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
STO 0	001, 44 0
7	002, 7
0	003, 0
3	004, 3
÷	005, 10
x≷y	006, 34
+	007, 40

12c platinum / 12C RPN KEYSTROKES	DISPLAY
[+]	008, 40
[RCL] 0	009, 45 0
[X]	010, 20
5	011, 5
6	012, 6
2	013, 2
[+]	014, 40
7	015, 7
0	016, 0
3	017, 3
[RCL] 0	018, 45 0
[÷]	019, 10
1	020, 1
6	021, 6
5	022, 5
[+]	023, 40
[÷]	024, 10
[CHS]	025, 16
[g] [e^x]	026, 43 22
2	027, 2
[÷]	028, 10
[g] [GTO] 000	029,43,33,000
[f] [P/R]	

12c platinum ALG KEYSTROKES	DISPLAY
1	008, 1
6	009, 6
5	010, 5
[=]	011, 36
[RCL] 0	012, 45 0
[X]	013, 20
8	014, 8
3	015, 3
[+]	016, 40
3	017, 3
5	018, 5
1	019, 1
[X]	020, 20
[RCL] 0	021, 45 0
[+]	022, 40
5	023, 5
6	024, 6
2	025, 2
[÷]	026, 10
[x≷y]	027, 34
[=]	028, 36
[g] [e^x]	029, 43 22
[+]	030, 40
[=]	031, 36
[1/x]	032, 22
[g] [GTO] 000	033,43,33,000
[f] [P/R]	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: x	R_1-R_6: Unused	

Program Instructions:

1. Key in the program.
2. Key in x and press [R/S] to compute $Q(x)$.
3. Repeat step 2 for each new case.

Example: Find $Q(x)$ for x = 1.18 and x = 2.1.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
1.18 [R/S]	1.18 [R/S]	**0.12**	*Q(1.18)*
2.1 [R/S]	2.1 [R/S]	**0.02**	*Q(2.1)*

Covariance

Covariance is a measure of the interdependence between paired variables (x and y). Like standard deviation, covariance may be defined for either a sample (S_{xy}) or a population (S'_{xy}) as follows:

$$S_{xy} = r \cdot s_x \cdot s_y$$
$$S'_{xy} = r \cdot s'_x \cdot s'_y$$

The following procedure finds the covariance of a sample (S_{xy}) and of a population (S'_{xy}):

1. Press [f] CLEAR [REG].
2. Key in the y-values and press [ENTER] ([=]).
3. Key in the x-values and press [Σ+].
4. Repeat steps 2 and 3 for all data pairs.

RPN Mode:

5. Press [g] [s] [X] [ENTER] [g] [ŷ,r] [R↓] [X] to obtain the value of S_{xy}.
6. Press [RCL] 1 1 [−] [RCL] 1 [÷] [X] to obtain S'_{xy}.

ALG Mode:

5. Press [g] [s] 1 [g] [ŷ,r] [R↓] [X] [x≷y] [X] [x≷y] [=] to obtain the value of S_{xy}.
6. Press 1 [−] [RCL] 1 [1/x] [X] [x≷y] [=] to obtain S'_{xy}.

Example 1: Find the sample covariance (S_{xy}) and population covariance (S'_{xy}) for the following paired variables:

x_i	26	30	44	50	62	68	74
y_i	92	85	78	81	54	51	40

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
[f] CLEAR [REG] 92 [ENTER] 26 [Σ+] 85 [ENTER] 30 [Σ+] 78 [ENTER] 44 [Σ+] 81 [ENTER] 50 [Σ+] 54 [ENTER] 62 [Σ+] 51 [ENTER] 68 [Σ+] 40 [ENTER] 74 [Σ+]	[f] CLEAR [REG] 92 [=] 26 [Σ+] 85 [=] 30 [Σ+] 78 [=] 44 [Σ+] 81 [=] 50 [Σ+] 54 [=] 62 [Σ+] 51 [=] 68 [Σ+] 40 [=] 74 [Σ+]	**7.00**	Total number of entries.
[g] [s] [X] [ENTER] [g] [ŷ,r] [R↓] [X]	[g] [s] 1 [g] [ŷ,r] [R↓] [X] [x≷y] [X] [x≷y] [=]	**-354.14**	S_{xy}
[RCL] 1 1 [−] [RCL] 1 [÷] [X]	1 [−] [RCL] 1 [1/x] [X] [x≷y] [=]	**-303.55**	S'_{xy}

Try the previous example using the following HP 12C Platinum program:

12c platinum / 12C RPN KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
Σ+	001, 49
g GTO 000	002,43,33,000
g s	003, 43 48
X	004, 20
ENTER	005, 36
g ŷ,r	006, 43 2
R↓	007, 33
X	008, 20
R/S	009, 31
RCL 1	010, 45 1
1	011, 1
−	012, 30
RCL	013, 45 1
÷	014, 10
X	015, 20
g GTO 000	016,43,33,000
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
Σ+	001, 49
g GTO 000	002,43,33,000
g s	003, 43 48
1	004, 1
g ŷ,r	005, 43 2
R↓	006, 33
X	007, 20
x≷y	008, 34
X	009, 20
x≷y	010, 34
=	011, 36
R/S	012, 31
1	013, 1
−	014, 30
RCL 1	015, 45 1
1/x	016, 22
X	017, 20
x≷y	018, 34
=	019, 36
g GTO 000	020,43,33,000
f P/R	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: Unused	R_1: n	R_2: Σx
R_3: Σx^2	R_4: Σy	R_5: Σy^2	R_6: Σxy
R_7-$R_{.7}$: Unused			

Program Instructions:

1. Key in the program.
2. Press f CLEAR REG.
3. Key in the y-value and press ENTER (=).
4. Key in the x-value and press R/S. Repeat steps 3 and 4 for all data pairs.
5. Press g GTO 003 R/S to obtain the value of S_{xy}.
6. Press R/S to obtain S'_{xy}.
7. For a new case, go to step 2.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
f CLEAR REG	f CLEAR REG		
92 ENTER 26 R/S	92 = 26 R/S		
85 ENTER 30 R/S	85 = 30 R/S		
78 ENTER 44 R/S	78 = 44 R/S		
81 ENTER 50 R/S	81 = 50 R/S		
54 ENTER 62 R/S	54 = 62 R/S		
51 ENTER 68 R/S	51 = 68 R/S		
40 ENTER 74 R/S	40 = 74 R/S	**7.00**	Total number of entries.
g GTO 003 R/S	g GTO 003 R/S	**-354.14**	S_{xy}
R/S	R/S	**-303.55**	S'_{xy}

Permutations

A permutation is an *ordered* subset of a set of distinct objects. The number of possible permutations, each containing n objects, that can be formed from a collection of m distinct objects is given by:

$$ {}_{m}P_{n} = \frac{m!}{(m-n)!} $$

where m, n are integers and $69 \geq m \geq n \geq 0$.

Use the following HP 12C Platinum program to calculate the number of possible permutations.

12c platinum / 12C RPN KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
STO 0	001, 44 0
x≷y	002, 34
g n!	003, 43 3
g LSTx	004, 43 40
RCL 0	005, 45 0
−	006, 30
g n!	007, 43 3
÷	008, 10
g GTO 000	009,43,33,000
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
g LSTx	001, 43 40
−	002, 30
x≷y	003, 34
=	004, 36
x≷y	005, 34
g n!	006, 43 3
÷	007, 10
x≷y	008, 34
g n!	009, 43 3
=	010, 36
g GTO 000	011,43,33,000
f P/R	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: *n*	R_1-R_8: Unused	

Program Instructions:

1. Key in the program.
2. Key in *m* and press ENTER (=).
3. Key in *n* and press R/S to calculate ${}_mP_n$.
4. For a new case go to step 2.

Example: How many ways can 10 people be seated on a bench if only 4 seats are available?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
10 ENTER 4 R/S	10 = 4 R/S	**5,040.00**	${}_{10}P_4$.

Combinations

A combination is a selection of one or more of a set of distinct objects without regard to order. The number of possible combinations, each containing *n* objects, that can be formed from a collection of *m* distinct objects is given by:

$$ {}_mC_n = \frac{m!}{(m-n)!n!} $$

Where *m*, *n* are integers and $69 \geq m \geq n \geq 0$.

Use the following HP 12C Platinum to calculate the number of possible combinations.

12c platinum / 12C RPN KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
STO 0	001, 44 0
x≷y	002, 34
g n!	003, 43 3
g LSTx	004, 43 40
RCL 0	005, 45 0
−	006, 30
g n!	007, 43 3
RCL 0	008, 45 0
g n!	009, 43 3
X	010, 20
÷	011, 10
g GTO 000	012,43,33,000
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
g LSTx	001, 43 40
−	002, 30
x≷y	003, 34
=	004, 36
x≷y	005, 34
g n!	006, 43 3
÷	007, 10
x≷y	008, 34
g n!	009, 43 3
÷	010, 10
g LSTx	011, 43 40
g n!	012, 43 3
=	013, 36
g GTO 000	014,43,33,000
f P/R	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: *n*	R_1-R_8: Unused	

Program Instructions:

1. Key in the program.
2. Key in *m* and press ENTER (=).
3. Key in *n* and press R/S to calculate ${}_mC_n$.
4. For a new case, go to step 2.

Example: A manager wants to choose a committee of three people from the seven engineers working for him. In how many different ways can the committee be selected?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
7 ENTER 3 R/S	7 = 3 R/S	**35.00**	${}_7C_3$.

Random Number Generator

This HP 12C Platinum program calculates uniformly distributed pseudo-random numbers u_i in the range

$$0 < u_i < 1$$

The following method is used:

- u_{i+1} = fractional part of (997 u_i)
- where i = 0, 1, 2, ...
- u_0 = 0.5284163* (seed)

The period of this generator has a length of 500,000 numbers and the generator passes the frequency test (chi square) for uniformity, the serial test and the run test. The most significant digits (the left hand digits) are the most random digits. The right most digits are significantly less random.

12c platinum / 12C RPN KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
•	001, 48
5	002, 5
2	003, 2
8	004, 8
4	005, 4
1	006, 1
6	007, 6
3	008, 3
STO 0	009, 44 0
9	010, 9
9	011, 9
7	012, 7
X	013, 20
g FRAC	014, 43 24
STO 0	015, 44 0
R/S	016, 31
g GTO 010	017,43,33,010
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
•	001, 48
5	002, 5
2	003, 2
8	004, 8
4	005, 4
1	006, 1
6	007, 6
3	008, 3
X	009, 20
9	010, 9
9	011, 9
7	012, 7
=	013, 36
g FRAC	014, 43 24
STO 0	015, 44 0
R/S	016, 31
g GTO 009	017,43,33,009
f P/R	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: U_i	R_1-R_7: Unused	

*Other seeds may be selected but the quotient of (seed x 10^7) divided by two or five must not be an integer. Also, it would be wise to statistically test other seeds before using them.

Program Instructions:

1. Key in the program.
2. To generate a random number, press R/S.
3. Repeat step 2 as many times as desired.

Example: Generate a sequence of 5 random numbers.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
R/S	R/S	**0.83**	Random number.
R/S	R/S	**0.56**	
R/S	R/S	**0.27**	
R/S	R/S	**0.04**	
R/S	R/S	**0.20**	

Personal Finance

Homeowners Monthly Payment Estimator

It is often useful, when comparison shopping for a mortgage or determining the appropriate price range of houses to consider, to be able to quickly estimate the monthly payment given the purchase price, tax rate per $1000, percent down, interest rate and term of the loan.

The calculation assumes that the assessed value is 100% of the sales price and does not take into account financing of the closing costs.

A simple keystroke procedure may be used to calculate the monthly payment:

1. Press [g] [END] and press [f] CLEAR [FIN].
2. Key in the annual interest rate and press [g] [12÷].
3. Key in the term of the loan (in years) and press [g] [12x].
4. Key in the purchase prices and press [STO] 1.

RPN Mode:

5. Key in the percent down and press [%] [−] [PV].
6. Key in the tax rate in dollars per thousand and press [RCL] 1 [X] 12000 [÷] [CHS] [ENTER] [PMT] [PMT] [+] .
   (A negative sign is the convention for cash paid out).

ALG Mode:

5. Press [−], key in the percent down and press [%] [PV] [PMT] [RCL] 1 [X].
6. Key in the tax rate in dollars per thousand and press [CHS] [÷] 12000 [+] [x≷y] [=].
   (A negative sign is the convention for cash paid out).

Example: What would your monthly payments be on a $65,000 house in a neighborhood with a $25 per thousand tax rate and a 10 ¾ % interest rate on a 35 year loan with 10% down?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
[g] [END] [f] CLEAR [FIN] 10.75 [g] [12÷]	[g] [END] [f] CLEAR [FIN] 10.75 [g] [12÷]	**0.90**	Monthly interest rate.
35 [g] [12x]	35 [g] [12x]	**420.00**	Months of loan.
65000 [STO] 1	65000 [STO] 1	**65,000.00**	Purchase price.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
10 [%] [−] [PV]	[−] 10 [%] [PV]	**58,500.00**	Mortgage balance.
25 [RCL] 1 [X] 12000 [÷] [CHS]	[PMT] [RCL] 1 [X] 25 [CHS] [÷] 12000 [+]	**-135.42**	Approximate monthly taxes.
[ENTER] [PMT] [PMT] [+]	[x≷y] [=]	**-672.16**	Approximate monthly payment.

The following HP 12C Platinum program may be used instead of the above.

12c platinum / 12C RPN KEYSTROKES	DISPLAY
[f] [P/R]	
[f] CLEAR [PRGM]	000,
[g] [END]	001, 43 8
[RCL] 1	002, 45 1
[RCL] 2	003, 45 2
[%]	004, 25
[−]	005, 30
[PV]	006, 13
[ENTER]	007, 36
[g] [LSTx]	008, 43 40
[+]	009, 40
[RCL] 3	010, 45 3
[X]	011, 20
1	012, 1
2	013, 2
[EEX]	014, 26
3	015, 3
[÷]	016, 10
[CHS]	017, 16
[ENTER]	018, 36
[PMT]	019, 14
[PMT]	020, 14
[+]	021, 40
[g] [GTO] 000	022,43,33,000
[f] [P/R]	

12c platinum ALG KEYSTROKES	DISPLAY
[f] [P/R]	
[f] CLEAR [PRGM]	000,
[g] [END]	001, 43 8
[RCL] 1	002, 45 1
[−]	003, 30
[RCL] 2	004, 45 2
[%]	005, 25
[PV]	006, 13
[PMT]	007, 14
[RCL] 1	008, 45 1
[X]	009, 20
[RCL] 3	010, 45 3
[÷]	011, 10
1	012, 1
2	013, 2
[EEX]	014, 26
3	015, 3
[−]	016, 30
[x≷y]	017, 34
[x≷y]	018, 34
[=]	019, 36
[g] [GTO] 000	020,43,33,000
[f] [P/R]	

	REGISTERS		
n: Term	i: Interest	PV: Loan	PMT: Loan PMT
FV: 0	R_0: Unused	R_1: Purch. Price	R_2: % Down
R_3: Tax rate	R_4-R_7: Unused		

Program Instructions:

1. Key in the program.
2. Press [f] CLEAR [FIN].
3. Key in the annual interest rate and press [g] [12÷] .
4. Key in the term of the loan in years and press [g] [12x] .
5. Key in the purchase price and press [STO] 1.
6. Key in the percent down and press [STO] 2.
7. Key in the tax rate in dollars per thousand and press [STO] 3.
8. To calculate the approximate monthly payment, press [R/S].
9. For a new case, store only the new variables by performing steps 3 thru 7 as needed. Press [R/S] for the new approximate monthly payment.

Example: Solve the previous example using the HP 12C Platinum program.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
[f] CLEAR [FIN] 10.75 [g] [12÷]	[f] CLEAR [FIN] 10.75 [g] [12÷]	**0.90**	Monthly interest.
35 [g] [12x]	35 [g] [12x]	**420.00**	Months of loan.
65000 [STO] 1	65000 [STO] 1	**65,000.00**	Purchase price.
10 [STO] 2	10 [STO] 2	**10.00**	Percent down.
25 [STO] 3	25 [STO] 3	**25.00**	Tax rate per thousand.
[R/S]	[R/S]	**-672.16**	Approximate monthly payment.

What would the approximate payment be if the loan was at 10% interest?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
10 [g] [12÷] [R/S]	10 [g] [12÷] [R/S]	**-638.33**	Approximate monthly payment.

What if the down payment is increased to 20%?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
20 [STO] 2 [R/S]	20 [STO] 2 [R/S]	**-582.45**	Approximate monthly payment.

Tax-Free Individual Retirement (IRA) or Keogh Plan

The advent of tax-free retirement accounts (IRA or Keogh) has resulted in considerable benefits for many persons who are not able to participate in group profit sharing or retirement plans. The savings due to the tax-free status are often considerable, but complex to calculate. Required data are: the years to retirement, the total annual investment, the compound annual interest rate of the investment, and an assumed tax rate (the *dividend tax rate*) which would be paid on a similar but taxable investment. This program calculates:

1. The future cash value of the tax-free investment (the *dividend tax rate* does not apply).
2. The total cash paid in.
3. The total dividends paid (the tax-free status means these dividends are tax-free).
4. The future value of the investment at retirement, assuming that after retirement you make withdrawals at a rate which causes the money to be taxed at the *withdrawal tax rate*. This rate is often assumed to be one half of the *dividend tax rate*.
5. The diminished purchasing power assuming a given annual inflation rate.
6. The future value of a comparable taxable investment (the *dividend tax rate* applies).
7. The diminished purchasing power of a comparable taxable investment.

Notes:

- The calculations run from the beginning of the first year to the end of the last year.
- The interest (annual yield), i, should be entered to as many significant figures as possible for maximum accuracy.

12c platinum / 12C RPN KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
RCL n	001, 45 11
RCL PMT	002, 45 14
X	003, 20
R/S	004, 31
+	005, 40
R/S	006, 31
RCL FV	007, 45 15
RCL 2	008, 45 2
%	009, 25
−	010, 30
R/S	011, 31
1	012, 1

12c platinum ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
RCL n	001, 45 11
X	002, 20
RCL PMT	003, 45 14
+	004, 40
R/S	005, 31
x≷y	006, 34
=	007, 36
R/S	008, 31
RCL FV	009, 45 15
−	010, 30
RCL 2	011, 45 2
%	012, 25

12c platinum / 12C RPN KEYSTROKES	DISPLAY
RCL 3	013, 45 3
%	014, 25
+	015, 40
RCL n	016, 45 11
y^x	017, 21
÷	018, 10
R/S	019, 31
RCL i	020, 45 12
RCL 1	021, 45 1
%	022, 25
−	023, 30
i	024, 12
FV	025, 15
R/S	026, 31
g GTO 012	027,43,33,012
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
=	013, 36
R/S	014, 31
1	015, 1
+	016, 40
RCL 3	017, 45 3
%	018, 25
=	019, 36
y^x	020, 21
RCL n	021, 45 11
÷	022, 10
x≷y	023, 34
x≷y	024, 34
=	025, 36
R/S	026, 31
RCL i	027, 45 12
−	028, 30
RCL 1	029, 45 1
%	030, 25
i	031, 12
FV	032, 15
R/S	033, 31
g GTO 015	034,43,33,015
f P/R	

	REGISTERS		
n: Years	i: Used	PV: 0	PMT: Yearly Pmt
FV: Used	R_0: Unused	R_1: Dividend Tax %	R_2: Withdrawal Tax %
R_3: Inflation %	R_4-R_5: Unused		

Program Instructions:

1. Key in the program.
2. Press f CLEAR REG and press g BEG.
3. Key in the dividend tax rate as a percentage and press STO 1.
3. Key in the withdrawal tax rate as a percentage and press STO 2.
3. Key in the inflation rate as a percentage and press STO 3.
6. Key in years to retirement and press n.
7. Key in the interest rates as a percentage and press i.
8. Key in the annual payment and press CHS PMT.
9. Press FV to calculate the future value of the tax free investment.
10. Press R/S to compute the total cash paid in.

11. Press [R/S] to compute the total dividends paid.
12. Press [R/S] to compute the future value when, after retirement, money is withdrawn at a rate causing the tax rate to equal ½ the rate paid during the pay in period.
13. Press [R/S] to compute the diminished purchasing power, in terms of today's dollars, of the future value assuming a 10% annual inflation rate.
14. Press [R/S] to compute the future value of an ordinary tax investment.
15. Press [R/S] to compute the diminished purchasing power of the ordinary tax investment.

Example: Assuming a 35 year investment period with a dividend rate of 8.175% and an income tax rate of 40%:

1. If you invest $1500 each year in a tax free account, what will its value be at retirement?
2. How much cash will be paid in?
3. What will be the value of the earned dividends?
4. After retirement, if you withdraw cash form the account at a rate such that it will be taxed at a rate equal to one-half the rate paid during the pay-in period, what will be the after-tax value?
5. What is the diminished purchasing power of that amount, in today's dollars, assuming 10% annual inflation?
6. If you invest the same amount ($1500 *after taxes* for a non-Keogh or non-IRA account) each year with dividends taxed as ordinary income, what will be the total tax-paid cash at retirement?
7. What is the purchasing power of that figure in terms of today's dollars?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
[f] CLEAR [REG] [g] [BEG] 40 [STO] 1	[f] CLEAR [REG] [g] [BEG] 40 [STO] 1	**40.00**	Dividend tax rate.
20 [STO] 2	20 [STO] 2	**20.00**	Withdrawal tax rate.
10 [STO] 3	10 [STO] 3	**10.00**	Inflation rate.
35 [n]	35 [n]	**35.00**	Years to retirement.
8.175 [i]	8.175 [i]	**8.18**	Dividend rate.
1500 [CHS] [PMT]	1500 [CHS] [PMT]	**-1,500.00**	Annual payment.
[FV]	[FV]	**290,730.34**	Future value at retirement.
[R/S]	[R/S]	**-52,500.00**	Cash paid in.
[R/S]	[R/S]	**238,230.34**	Earned dividends (untaxed).
[R/S]	[R/S]	**232,584.27**	After-tax value.
[R/S]	[R/S]	**8,276.30**	Diminished purchasing power.
[R/S]	[R/S]	**139,360.09**	Tax-paid cash at retirement.
[R/S]	[R/S]	**4,959.00**	Purchasing power of tax-paid cash at retirement.

Stock Portfolio Evaluation and Analysis

This program evaluates a portfolio of stocks given the current market price per share and the annual dividend. The user inputs the initial purchase price of a stock, the number of shares, the beta coefficient*, the annual dividend, and the current market price for a portfolio of any size.

The program returns the percent change in value of each stock and the valuation and beta coefficient* of the entire portfolio. Output includes the original portfolio value, the new portfolio value, the percent change in the value and the annual dividend and yield as a percent of the current market value. The overall beta coefficient of the portfolio is also calculated.

Note:

The beta coefficient analysis is optional. Key in 1.00 if beta is not to be analyzed.

12c platinum / 12C RPN KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
RCL 4	001, 45 4
g x=0	002, 43 35
g GTO 021	003,43,33,021
STO − 4	004,44 30 4
X	005, 20
RCL 7	006, 45 7
X	007, 20
STO + 0	008,44 40 0
x≷y	009, 34
RCL 7	010, 45 7
X	011, 20
STO + 1	012,44 40 1
R↓	013, 33
X	014, 20
STO + 3	015,44 40 3
RCL 5	016, 45 5
g LSTx	017, 43 40
Δ%	018, 24
R/S	019, 31

12c platinum ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
STO 6	001, 44 6
RCL 4	002, 45 4
g x=0	003, 43 35
g GTO 026	004,43,33,026
STO − 4	005,44 30 4
R↓	006, 33
X	007, 20
RCL 7	008, 45 7
=	009, 36
STO + 0	010,44 40 0
x≷y	011, 34
X	012, 20
RCL 7	013, 45 7
=	014, 36
STO + 1	015,44 40 1
R↓	016, 33
X	017, 20
x≷y	018, 34
=	019, 36

* The beta coefficient is a measure of a stock variability (risk) compared to the market in general. Beta values for individual stocks can be acquired from brokers, investment publications or the local business library.

12c platinum / 12C RPN KEYSTROKES	DISPLAY
g GTO 001	020,43,33,001
+	021, 40
x≷y	022, 34
STO 7	023, 44 7
X	024, 20
STO 5	025, 44 5
STO + 2	026,44 40 2
1	027, 1
STO 4	028, 44 4
R/S	029, 31
g GTO 001	030,43,33,001
RCL 2	031, 45 2
R/S	032, 31
RCL 0	033, 45 0
R/S	034, 31
Δ%	035, 24
R/S	036, 31
RCL 0	037, 45 0
RCL 1	038, 45 1
R/S	039, 31
%T	040, 23
R/S	041, 31
RCL 3	042, 45 3
RCL 0	043, 45 0
÷	044, 10
g GTO 000	045,43,33,000
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
STO + 3	020,44 40 3
RCL 5	021, 45 5
RCL 6	022, 45 6
Δ%	023, 24
R/S	024, 31
g GTO 001	025,43,33,001
R↓	026, 33
STO 5	027, 44 5
X	028, 20
x≷y	029, 34
STO 7	030, 44 7
=	031, 36
STO + 2	032,44 40 2
1	033, 1
STO 4	034, 44 4
R/S	035, 31
g GTO 001	036,43,33,001
RCL 2	037, 45 2
R/S	038, 31
RCL 0	039, 45 0
R/S	040, 31
Δ%	041, 24
R/S	042, 31
RCL 0	043, 45 0
RCL 1	044, 45 1
R/S	045, 31
%T	046, 23
R/S	047, 31
RCL 3	048, 45 3
÷	049, 10
RCL 0	050, 45 0
=	051, 36
g GTO 000	052,43,33,000
f P/R	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: ΣPV	R_1: ΣDIV	R_2: ΣOrig. Val.
R_3: $\Sigma P_i S_i \beta_i$	R_4: Flag	R_5: $P_i\, n_i$	R_6: P_i
R_7: S_i	R_8-$R_{.1}$: Unused		

Program Instructions:

1. Key in the program.
2. Initialize the program by pressing f CLEAR REG.
3. Key in the number of shares of a stock and press ENTER (=).
4. Key in the initial purchase of the stock and press R/S.
5. Key in the beta coefficient of the stock and press ENTER (=).
6. Key in the annual dividend of the stock and press ENTER (=).
7. Key in the present price of the stock and press R/S. The display will show the percent change in the stock value.
8. Repeat steps 3 through 7 until all the stocks are entered.
9. Next, to evaluate the entire portfolio, press :

RPN: g GTO 031
ALG: g GTO 037

10. Press R/S to see the initial portfolio value.
11. Press R/S to see the present portfolio value.
12. Press R/S to see the percent change in value.
13. Press R/S to see the total yearly dividend.
14. Press R/S to see the annual dividend yield as a percent of the current market value.
15. Press R/S to see the beta coefficient of the portfolio.
16. For a new case return to step 2.

Example: Evaluate the following portfolio:

Number of Shares Held	Initial Purchase Price	Beta Coefficient	Annual Dividend	Present Market Price	Stock
100	25.63	.8	$1.70	27.25	Int'l Heartburn
200	30.25	1.2	$2.10	33.50	P. D. Q.
50	89.88	1.3	$4.55	96.13	Datacrunch
500	65.25	.6	$3.50	64.38	N.W. Sundial

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
f CLEAR REG	f CLEAR REG	**0.00**	Int'l Heartburn
100 ENTER	100 =	**100.00**	
25.63 R/S	25.63 R/S	**1.00**	
.8 ENTER	.8 =	**0.80**	
1.70 ENTER	1.70 =	**1.70**	
27.25 R/S	27.25 R/S	**6.32**	Percent change in Stock's value.

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
200 ENTER 30.25 R/S 1.2 ENTER 2.10 ENTER 33.5 R/S	200 = 30.25 R/S 1.2 = 2.10 = 33.5 R/S	**200.00** **1.00** **1.20** **2.10** **10.74**	P. D. Q. Percent change in Stock's value.
50 ENTER 89.88 R/S 1.3 ENTER 4.55 ENTER 96.13 R/S	50 = 89.88 R/S 1.3 = 4.55 = 96.13 R/S	**50.00** **1.00** **1.30** **4.55** **6.95**	Datacrunch Percent change in Stock's value.
500 ENTER 65.25 R/S .6 ENTER 3.50 ENTER 64.38 R/S	500 = 65.25 R/S .6 = 3.50 = 64.38 R/S	**500.00** **1.00** **0.60** **3.50** **-1.33**	N. W. Sundial Percent change in Stock's value.
g GTO 031 R/S	g GTO 037 R/S	 **45,732.00**	 Original value.
R/S	R/S	**46,421.50**	Present value.
R/S	R/S	**1.51**	Percent change in value.
R/S	R/S	**2,567.50**	Total yearly dividend.
R/S	R/S	**5.53**	Annual dividend yield.
R/S	R/S	**0.77**	Portfolio beta coefficient.

Canadian Mortgages

In Canada, interest is compounded semi-annually with payments made monthly. This results in a different monthly mortgage factor than is used in the United States and preprogrammed into the HP 12C Platinum. This difference can be easily handled by the addition of a few keystrokes. For any problem requiring an input for [i], the Canadian mortgage factor is calculated first and then this value is entered in for [i] in the calculation to give the answer for Canada.

The keystrokes to calculate the monthly Canadian mortgage factor are:

RPN Mode:

1. Press [f] CLEAR [FIN] [g] [END].
2. Key in 6 and press [n].
3. Key in 200 and press [ENTER] [PV].
4. Key in the annual interest rate as a percentage and press [+] [CHS] [FV].
5. Press [i].

ALG Mode:

1. Press [f] CLEAR [FIN] [g] [END].
2. Key in 2 and press [n].
3. Key in the annual interest rate as a percentage and press [÷] 2 [i].
4. Key in 1 and press [PV] [FV].
5. Press 12 [n] [i].

The Canadian mortgage factor is now stored in [i] for future use. The examples below show how this factor is used for [i] in Canadian mortgage problems. Example 3 shows how to reverse this procedure and obtain the annual Canadian interest rate from the monthly Canadian mortgage factor.

Periodic Payment Amount

Example 1: What is the monthly payment required to fully amortize a 30-year, $30,000 Canadian mortgage if the interest rate is 9%?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
[f] CLEAR [FIN] [g] [END] 6 [n] 200 [ENTER] [PV] 9 [+] [CHS] [FV] [i]	[f] CLEAR [FIN] [g] [END] 2 [n] 9 [÷] 2 [i] 1 [PV] [FV] 12 [n] [i]	**0.74**	Canadian mortgage factor.
30 [g] [12x]	30 [g] [12x]	**360.00**	Total monthly periods in mortgage life.
30000 [PV] 0 [FV] [PMT]	30000 [PV] 0 [FV] [PMT]	**-237.85**	Monthly payment.

Number of Periodic Payments to Fully Amortize a Mortgage

Example 2: An investor can afford to pay $440 per month on a $56,000 Canadian mortgage. If the annual interest rate is 9 ¼ %, how long will it take to completely amortize this mortgage?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
f CLEAR FIN g END 6 n 200 ENTER PV 9.25 + CHS FV i	f CLEAR FIN g END 2 n 9.25 ÷ 2 i 1 PV FV 12 n i	**0.76**	Canadian mortgage factor.
440 CHS PMT	440 CHS PMT	**-440.00**	Monthly payment.
56000 PV 0 FV n	56000 PV 0 FV n	**437.00**	Total number of monthly payments.

Effective Interest Rate (Yield)

Example 3: A Canadian mortgage has monthly payments of $612.77 with a maturity of 25 years. The principal amount is $75,500. What is the annual interest rate?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
f CLEAR FIN g END 25 g 12x 612.77 CHS PMT 75500 PV i	f CLEAR FIN g END 25 g 12x 612.77 CHS PMT 75500 PV i	**0.72**	Canadian mortgage factor.
6 n 0 PMT 200 CHS PV FV RCL PV +	0 PMT 12 n FV 2 n i X 2 =	**8.75**	Annual interest rate.

Balance Remaining at End of Specified Period

Example 4: A Canadian mortgage has monthly payments of $612.77 at 8.75% interest. The principal amount is $75,500. What will be the outstanding balance remaining at the end of 10 years?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
f CLEAR FIN g END 6 n 200 ENTER PV 8.75 + CHS FV i	f CLEAR FIN g END 2 n 8.75 ÷ 2 i 1 PV FV 12 n i	**0.72**	Canadian mortgage factor.
612.77 CHS PMT 10 g 12x 75500 PV FV	612.77 CHS PMT 10 g 12x 75500 PV FV	**-61,877.18**	Outstanding balance remaining at the end of 10 years.

Miscellaneous

Learning Curve for Manufacturing Costs

Many production process costs vary with output according to the "learning curve" equation. The production team becomes more proficient in manufacturing a given item as more and more of them are fabricated and costs may be expected to decrease by a predictable amount. The learning factor, *r*, characterizes the learning curve. For instance, if *r* =.80 the curve is called an 80% learning curve.

It is readily apparent that the learning, or experience curve, has many uses in setting production standards, forecasting costs, setting prices, etc. Note, however, that the learning factor may change, especially after large numbers have been produced.

It the cost of the first unit of a run, C_1, and the learning curve factor, *r*, are known, the following procedure can be used to calculate the cost of the nth item:

1. Key in the cost of the first item, C_1 and press [ENTER]([=]).
2. Key in the number of units produced, *n*, and press [ENTER]([=]).

RPN Mode:

3. Key in the learning factor, *r*, and press [g][LN]2[g][LN][÷].
4. Then press [y^x][X] to calculate the cost of the nth unit, *Cn*.

ALG Mode:

3. Key in the learning factor, *r*, and press [g][LN][÷]2[g][LN][y^x].
4. Then press [x≷y][x≷y][X][x≷y][=]to calculate the cost of the nth unit, *Cn*.

Example 1: An electronic manufacturer begins a pilot run on a new instrument. From past experience he expects the process to have a learning factor, *r*, or 0.90. If the first unit costs $875 to produce, what is the expected cost of the 100th unit?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
875[ENTER]	875[=]	**875.00**	
100[ENTER]	100[=]	**100.00**	
.9[g][LN]2[g][LN][÷]	.9[g][LN][÷]2[g][LN][y^x]	**-0.15**	
[y^x][X]	[x≷y][x≷y][X][x≷y][=]	**434.51**	Cost of the 100th unit.

If the cost of the first unit, C_1, and the *n*th unit, C_n, are known the learning factor may be calculated. In addition, it is possible to calculate C_{ij}, the average cost of the *i*th thru *j*th

unit. These calculations may be rapidly done with the following HP 12C Platinum program:

12c platinum / 12C RPN KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
g LN	001, 43 23
2	002, 2
g LN	003, 43 23
÷	004, 10
STO 2	005, 44 2
R↓	006, 33
x≷y	007, 34
STO 1	008, 44 1
÷	009, 10
g LN	010, 43 23
RCL 2	011, 45 2
÷	012, 10
g e^x	013, 43 22
STO 2	014, 44 2
g GTO 000	015,43,33,000
RCL 2	016, 45 2
g LN	017, 43 23
2	018, 2
g LN	019, 43 23
÷	020, 10
y^x	021, 21
RCL 1	022, 45 1
X	023, 20
g GTO 000	024,43,33,000
STO 3	025, 44 3
x≷y	026, 34
STO 4	027, 44 4
RCL 2	028, 45 2
g LN	029, 43 23
2	030, 2
g LN	031, 43 23
÷	032, 10
1	033, 1
+	034, 40
STO 0	035, 44 0
y^x	036, 21
RCL 3	037, 45 3
RCL 0	038, 45 0

12c platinum ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
g LN	001, 43 23
÷	002, 10
2	003, 2
g LN	004, 43 23
=	005, 36
STO 2	006, 44 2
R↓	007, 33
÷	008, 10
x≷y	009, 34
STO 1	010, 44 1
=	011, 36
g LN	012, 43 23
÷	013, 10
RCL 2	014, 45 2
=	015, 36
g e^x	016, 43 22
STO 2	017, 44 2
g GTO 000	018,43,33,000
RCL 2	019, 45 2
g LN	020, 43 23
÷	021, 10
2	022, 2
g LN	023, 43 23
=	024, 36
x≷y	025, 34
y^x	026, 21
x≷y	027, 34
X	028, 20
RCL 1	029, 45 1
=	030, 36
g GTO 000	031,43,33,000
STO 3	032, 44 3
x≷y	033, 34
STO 4	034, 44 4
RCL 2	035, 45 2
g LN	036, 43 23
÷	037, 10
2	038, 2

12c platinum / 12C RPN KEYSTROKES	DISPLAY
[y^x]	039, 21
[−]	040, 30
[RCL] 0	041, 45 0
[÷]	042, 10
[RCL] 4	043, 45 4
[RCL] 3	044, 45 3
[−]	045, 30
[÷]	046, 10
[RCL] 1	047, 45 1
[X]	048, 20
[g] [GTO] 000	049,43,33,000
[f] [P/R]	

12c platinum ALG KEYSTROKES	DISPLAY
[g] [LN]	039, 43 23
[+]	040, 40
1	041, 1
[=]	042, 36
[STO] 0	043, 44 0
[x≷y]	044, 34
[y^x]	045, 21
[x≷y]	046, 34
[=]	047, 36
[RCL] 3	048, 45 3
[y^x]	049, 21
[RCL] 0	050, 45 0
[=]	051, 36
[x≷y]	052, 34
[−]	053, 30
[x≷y]	054, 34
[÷]	055, 10
[RCL] 0	056, 45 0
[=]	057, 36
[RCL] 4	058, 45 4
[−]	059, 30
[RCL] 3	060, 45 3
[=]	061, 36
[x≷y]	062, 34
[÷]	063, 10
[x≷y]	064, 34
[X]	065, 20
[RCL] 1	066, 45 1
[=]	067, 36
[f] [P/R]	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: K+1	R_1: C_1	R_2: r
R_3: i	R_4: j	R_5-R_3: Unused	

Program Instructions:

1. Key in the program.

   Note: If the average costs are not going to be calculated:

RPN: lines 25 through 48 need not be keyed in
ALG: lines 32 through 67 need not be keyed in

2. To calculate *r*, the learning factor, if C_1 and C_n are known:
   a. Key in C_1, the cost of the first unit and press ENTER (=).
   b. Key in C_n, the cost of the *n*th unit and press ENTER (=).
   c. Key in *n*, the number of units and press R/S to calculate *r*, the learning factor.
3. To calculate the cost of the *n*th unit when C_1 and *r* are known:
   a. Key in C_1 and press STO 1. Key in *r* and press STO 2. (Note: This step may be skipped if step 2 has just been done).
   b. Key in the number of units, *n* and calculate C_n, the cost of the *n*th unit by pressing

RPN: g GTO 016 R/S.
ALG: g GTO 019 R/S.

4. To calculate the average cost per unit of the *i*th through *j*th unit, C_{ij}, if C_1 and *r* are known.
   a. Key in C_1 and press STO 1. Key in *r* and press STO 2. (Note: This step may be skipped if step 2 has just been done).
   b. Key in the number of the last unit of the batch, *j* and press ENTER (=).
   c. Key in the number of the first unit of the batch, *i*, and calculate the average cost per unit by pressing

RPN: g GTO 025 R/S.
ALG: g GTO 032 R/S.

Example 2: The electronic manufacturer cited in example 1 found that the 100th instrument actually cost $395 to manufacture. Find the actual learning factor, *r*, the cost of the 500th unit and the average cost of units 500 thru 1000. (Recall that C_1 was $875).

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
875 ENTER 395 ENTER 100 R/S	875 = 395 = 100 R/S	**875.00** **395.00** **0.89**	Actual *r*.
500 g GTO 016 R/S	500 g GTO 019 R/S	**299.14**	Cost of the 500th unit.
1000 ENTER 500 g GTO 025 R/S	1000 = 500 g GTO 032 R/S	**1000.00** **280.00**	Average cost of the 500th thru 1000th unit.

Queuing and Waiting Theory

Waiting lines, or queues, cause problems in many marketing situations. Customer goodwill, business efficiency, labor and space considerations are only some of the problems which may be minimized by proper application of queuing theory.

Although queuing theory can be complex and complicated subject, handheld calculators can be used to arrive at helpful decisions.

One common situation that we can analyze involves the case of several identical stations serving customers, where the customers arrive randomly in unlimited numbers. Suppose there are n (1 or more) identical stations serving the customers. λ is the arrival rate (Poisson input) and μ is the service rate (exponential service). We will assume that all customers are served on a first-come, first-served basis and wait in a single line (queue) then are directed to whichever station is available. We also will assume that no customers are lost from the queue. This situation, for instance, would be closely approximated by customers at some banking operations.

The formulas for calculating some of the necessary probabilities are too complex for simple keystroke solution. However, tables listing these probabilities are available and can be used to aid in quick solutions. Using the assumptions outlined above and a suitable table giving mean waiting time as a multiple of mean service (see page 512 of the Reference) the following keystroke solutions may be obtained:

RPN Mode:

1. Key in the arrival rate of customers, λ, and press [ENTER].
2. Key in the service rate, μ, and press [÷] to calculate ρ, the **intensity factor**. (Note ρ must be less than n for valid results, otherwise the queue will lengthen without limit).
3. Key in n, the number of servers and press [÷] to calculate ρ/n.
4. For a given n and ρ/n find the mean waiting time as a multiple of mean service time from the table. Key it in and press [ENTER].
5. Calculate the **average waiting time** in the queue by keying in the service rate, μ, and pressing [STO] 1 [÷] [STO] 2.
6. Calculate the **average waiting time** in the system by pressing [RCL] 1 [1/x] [+].
7. Key in λ and press [RCL] 2 [X] to calculate the **average queue length**.
8. Key in ρ, the intensity factor (from step 2 above) and press [+] to calculate the **average number of customers in the system**.

ALG Mode:

1. Key in the arrival rate of customers, λ, and press [÷].
2. Key in the service rate, μ, and press [÷] to calculate ρ, the **intensity factor**. (Note ρ must be less than n for valid results, otherwise the queue will lengthen without limit).
3. Key in n, the number of servers and press [=] to calculate ρ/n.
4. For a given n and ρ/n find the mean waiting time as a multiple of mean service time from the table. Key it in and press [÷].
5. Calculate the **average waiting time** in the queue by keying in the service rate, μ, and pressing [STO] 1 [+] [STO] 2.
6. Calculate the **average waiting time** in the system by pressing [RCL] 1 [1/x] [=].
7. Key in λ and press [×] [RCL] 2 [+] to calculate the **average queue length**.
8. Key in ρ, the intensity factor (from step 2 above) and press [=] to calculate the **average number of customers in the system**.

Reference:

Richard E Trueman, "An Introduction to Quantitative Methods for Decision Making," Holt, Rinehart and Winston, New York, 1977

Example 1: Bank customers arrive at a bank on an average of 1.2 customers per minute. They join a common queue for three tellers. Each teller completes a transaction at the rate of one customer every 2 minutes (0.5 customers per minute). What is the average waiting time in the queue? In the system? What is the average number of customers in the queue? In the system?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
1.2 [ENTER] .5 [÷]	1.2 [÷] .5 [÷]	**1.20** **2.40**	ρ, intensity factor.
3 [÷]	3 [=]	**0.80**	ρ / n

From Table 12.2, page 512 of the reference, the mean waiting time as a multiple of mean service time for n = 3, ρ/n = 0.8 is 1.079. (Note S is used instead of n in the reference's notation).

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
1.079 [ENTER] .5 [STO] 1 [÷] [STO] 2	1.079 [÷] .5 [STO] 1 [+] [STO] 2	**1.08** **2.16**	Average wait in queue (min).
[RCL] 1 [1/x] [+]	[RCL] 1 [1/x] [=]	**4.16**	Average wait in system (min).
1.2 [RCL] 2 [×]	1.2 [×] [RCL] 2 [+]	**2.59**	Average queue length.
2.4 [+]	2.4 [=]	**4.99**	Average # of customers in system.

If the number of servers is limited to one, with other conditions remaining the same (unlimited queue, Poisson arrival, exponential service), the average queue length can be readily calculated without reference to tables:

RPN Mode:

1. Key in the arrival rate, λ and press STO 1.
2. Key in the service rate, μ and press STO 2 ÷ ENTER ENTER 2 y^x x≷y 1 x≷y − ÷ to calculate the average number of customers waiting in queue at any one time.
3. Press RCL 1 ÷ to calculate the average waiting time.
4. Press RCL 2 1/x + to calculate the average total time the customer spends in the system.
5. Press RCL 1 X to calculate the average number of customers in the system.

ALG Mode:

1. Key in the arrival rate, λ and press STO 1 ÷.
2. Key in the service rate, μ and press STO 2 = 1/x − g x^2 x≷y = 1/x to calculate the average number of customers waiting in queue at any one time.
3. Press ÷ RCL 1 + to calculate the average waiting time.
4. Press RCL 2 1/x X to calculate the average total time the customer spends in the system.
5. Press RCL 1 = to calculate the average number of customers in the system.

Example 2: A small grocery store has but a single check-out counter. Customers arrive at a rate of 1 every 2 minutes (λ = .5) and, on the average, customers can be checked out at a rate of .9 per minute (μ). What is the average number of customers in the waiting line at any time? The average waiting time? What is the average total time for a customer to wait and be checked out? The average number of customers in the system?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
.5 STO 1 .9 STO 2 ÷ ENTER ENTER 2 y^x x≷y	.5 STO 1 ÷ .9 STO 2 =	**0.50** **0.56**	 (intensity factor, ρ)
1 x≷y − ÷	1/x − g x^2 x≷y = 1/x	**0.69**	Average # customers waiting in queue.
RCL 1 ÷	÷ RCL 1 +	**1.39**	Average waiting time.
RCL 2 1/x +	RCL 2 1/x X	**2.50**	Average total time in the system.
RCL 1 X	RCL 1 =	**1.25**	Average # customers in system.

With an HP 12C Platinum program one can readily calculate the necessary probabilities for this type of problem (dispensing with the use of tables) and perform additional calculations as well.

12c platinum / 12C RPN KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
1	001, 1
STO − 0	002,44 33 0
RCL • 0	003,45 48 0
RCL 0	004, 45 0
0	005, 0
g x≤y	006, 43 34
g GTO 009	007,43,33,009
g GTO 016	008,43,33,016
+	009, 40
y^x	010, 21
g LSTx	011, 43 40
g n!	012, 43 3
÷	013, 10
Σ+	014, 49
g GTO 001	015,43,33,001
RCL • 0	016,45 48 0
RCL 7	017, 45 7
y^x	018, 21
1	019, 1
RCL • 0	020,45 48 0
RCL 7	021, 45 7
÷	022, 10
−	023, 30
÷	024, 10
RCL 7	025, 45 7
g n!	026, 43 3
÷	027, 10
STO 6	028, 44 6
RCL 2	029, 45 2
+	030, 40
1/x	031, 22
STO 1	032, 44 1
RCL 6	033, 45 6
X	034, 20
STO 2	035, 44 2
RCL • 0	036,45 48 0

12c platinum ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
1	001, 1
STO − 0	002,44 30 0
RCL 0	003, 45 0
0	004, 0
g x≤y	005, 43 34
g GTO 008	006,43,33,008
g GTO 017	007,43,33,017
RCL • 0	008,45 48 0
y^x	009, 21
RCL 0	010, 45 0
÷	011, 10
RCL 0	012, 45 0
g n!	013, 43 3
=	014, 36
Σ+	015, 49
g GTO 001	016,43,33,001
RCL • 0	017,45 48 0
÷	018, 10
RCL 7	019, 45 7
−	020, 30
1	021, 1
=	022, 36
CHS	023, 16
RCL • 0	024,45 48 0
y^x	025, 21
RCL 7	026, 45 7
÷	027, 10
x≥y	028, 34
÷	029, 10
RCL 7	030, 45 7
g n!	031, 43 3
=	032, 36
STO 6	033, 44 6
+	034, 40
RCL 2	035, 45 2
=	036, 36

12c platinum / 12C RPN KEYSTROKES	DISPLAY
X	037, 20
RCL 7	038, 45 7
RCL • 0	039,45 48 0
−	040, 30
÷	041, 10
STO 3	042, 44 3
RCL • 0	043,45 48 0
+	044, 40
STO 4	045, 44 4
RCL 8	046, 45 8
÷	047, 10
STO 5	048, 44 5
RCL 3	049, 45 3
RCL 8	050, 45 8
÷	051, 10
STO 6	052, 44 6
R/S	053, 31
RCL 8	054, 45 8
RCL 7	055, 45 7
RCL 9	056, 45 9
X	057, 20
−	058, 30
X	059, 20
g e^x	060, 43 22
RCL 2	061, 45 2
X	062, 20
g GTO 053	063,43,33,053
f P/R	

12c platinum ALG KEYSTROKES	DISPLAY
1/x	037, 22
STO 1	038, 44 1
STO 2	039, 44 2
RCL 6	040, 45 6
STO X 2	041,44 20 2
RCL 7	042, 45 7
−	043, 30
RCL • 0	044,45 48 0
=	045, 36
RCL • 0	046,45 48 0
÷	047, 10
x≷y	048, 34
X	049, 20
RCL 2	050, 45 2
+	051, 40
STO 3	052, 44 3
RCL • 0	053,45 48 0
÷	054, 10
STO 4	055, 44 4
RCL 8	056, 45 8
=	057, 36
STO 5	058, 44 5
RCL 3	059, 45 3
÷	060, 10
RCL 8	061, 45 8
=	062, 36
STO 6	063, 44 6
R/S	064, 31
RCL 7	065, 45 7
X	066, 20
RCL 9	067, 45 9
−	068, 30
RCL 8	069, 45 8
X	070, 20
x≷y	071, 34
=	072, 36
CHS	073, 16
g e^x	074, 43 22
X	075, 20
RCL 2	076, 45 2
=	077, 36
g GTO 064	078,43,33,064
f P/R	

	REGISTERS		
n: Unused	i: Unused	PV: Unused	PMT: Unused
FV: Unused	R_0: K	R_1: P_0	R_2: P_b
R_3: L_q	R_4: L	R_5: T	R_6: Used, T_q
R_7: n	R_8: λ	R_9: μ	$R_{.0}$: ρ
$R_{.1}$: Unused			

Program Instructions:

1. Key in the program and press [f] CLEAR [REG].
2. Key in the number of servers, n and press [STO] 0 [STO] 7.

RPN Mode:

3. Key in the arrival rate of customers, λ and press [STO] 8.
4. Key in the service rate of each server, μ and press [STO] 9.
5. Press [÷] [STO] [•] 0 to calculate and store ρ the intensity factor.

ALG Mode:

3. Key in the arrival rate of customers, λ and press [STO] 8 [÷].
4. Key in the service rate of each server, μ and press [STO] 9.
5. Press [=] [STO] [•] 0 to calculate and store ρ the intensity factor.

6. Press [R/S] to see T_q, the average waiting time in the queue. Display P_0, probability that all servers are idle, by pressing [RCL] 1. Display P_b, probability that all servers are busy by pressing [RCL] 2. Display L_q, average number waiting in the queue by pressing [RCL] 3. Display L, the average number in the system (waiting and being served), by pressing [RCL] 4.

   Display T, average total time through the system, by pressing [RCL] 5. T_q, the average waiting time in the queue, may again be displayed by pressing [RCL] 6.
7. If desired, calculate P(t), the probability of waiting longer than a given time, by keying in the time and pressing [R/S].
8. Repeat step 7 for other times of interest.

Example 3: Using the data from example 1 of the keystroke solutions verify the data obtained. In addition, obtain P_0, the probability that none of the tellers are busy, and P_b the probability that all the tellers are busy. What is the probability that a customer will have to wait 2 minutes or more?

12c platinum / 12C RPN Keystrokes	12c platinum ALG Keystrokes	Display	Comments
f CLEAR REG	f CLEAR REG	**0.00**	
3 STO 0 STO 7	3 STO 0 STO 7	**3.00**	n
1.2 STO 8	1.2 STO 8 ÷	**1.20**	λ
.5 STO 9	.5 STO 9	**0.50**	μ
÷ STO • 0	= STO • 0	**2.40**	ρ
R/S	R/S	**2.16**	T_q average waiting time in queue.
RCL 1	RCL 1	**0.06**	P_0 probability all servers are idle.
RCL 2	RCL 2	**0.65**	P_b probability all servers are busy.
RCL 3	RCL 3	**2.59**	L_q average # waiting in queue.
RCL 4	RCL 4	**4.99**	L, average # waiting in system.
RCL 5	RCL 5	**4.16**	T, average total time in system.
2 R/S	2 R/S	**0.36**	Probability of having to wait 2 minutes or more.

Appendix A

Algebraic Versions of Programs from Part III of the Owner's Handbook

About this Appendix

This appendix contains algebraic versions of the eleven programs found in Part III of the *HP12C Platinum Owner's Handbook and Problem-Solving Guide.* It does not cover the step-by-step examples given in Part III.

These programs are solely for the HP 12C Platinum, and work properly only when the calculator is set to ALG mode.

They should be tested by running the corresponding program examples given in the *HP12C Platinum Owner's Handbook and Problem-Solving Guide.* They work exactly the same as the RPN versions. The instructions for running these programs are not included here. They are simply listed with reference to the relevant section in the *HP12C Platinum Owner's Handbook and Problem-Solving Guide.* The register usage is the same as in the RPN versions.

In order to execute miscellaneous side calculations which are done during the program examples, these additional steps are required in algebraic mode:

1. The first three depreciation programs specify a side calculation to calculate the total depreciation through the current year - the algebraic version of this is: [RCL] [PV] [+] [RCL] 3 [−] [x≷y] [−] [RCL] [FV] [=].
2. Section 14 - the *Advance Payments with Residual-Solving for Payment* program *Example 1* stores 15/12 in R_1 and *Example 2* stores 18/12 in R_1. The algebraic keystrokes are: 15 [÷] 12 [=] [STO] 1 and 18 [÷] 12 [=] [STO] 1.
3. At the end of the bond programs the total price is obtained by pressing: [+] [x≷y] [=].
4. Section 16 - the *30/360 Day Basis Bonds* program *Example 2* stores 93 ⅜ in R_1. The algebraic keystrokes are: 3 [÷] 8 [+] 93 [=] [STO] 1.

The program listings have been formatted in double columns and apart from one instance the listings do not span pages, thus making key entry convenient.

Algebraic Mode Programs

Section 12: The Rent or Buy Decision

ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
FV	001, 15
FV	002, 15
RCL n	003, 45 11
STO 0	004, 44 0
RCL 2	005, 45 2
g 12x	006, 43 11
RCL 3	007, 45 3
g 12÷	008, 43 12
RCL PV	009, 45 13
−	010, 30
RCL 1	011, 45 1
PV	012, 13
RCL FV	013, 45 15
0	014, 0
FV	015, 15
PMT	016, 14
R↓	017, 33
RCL 0	018, 45 0
g 12x	019, 43 11
f AMORT	020, 42 11
x≷y	021, 34
R↓	022, 33
÷	023, 10
x≷y	024, 34
−	025, 30
RCL 4	026, 45 4
x	027, 20
RCL • 0	028,45 48 0
%	029, 25
=	030, 36
RCL PMT	031, 45 14

ALG KEYSTROKES	DISPLAY
−	032, 30
RCL 4	033, 45 4
−	034, 30
RCL 5	035, 45 5
+	036, 40
RCL 8	037, 45 8
−	038, 30
x≷y	039, 34
PMT	040, 14
R↓	041, 33
CHS	042, 16
−	043, 30
RCL 7	044, 45 7
%	045, 25
−	046, 30
RCL PV	047, 45 13
FV	048, 15
R/S	049, 31
RCL 0	050, 45 0
g 12x	051, 43 11
RCL 1	052, 45 1
CHS	053, 16
−	054, 30
RCL 6	055, 45 6
PV	056, 13
i	057, 12
RCL g 12÷	058,45,43 12
R/S	059, 31
RCL 9	060, 45 9
g 12÷	061, 43 12
FV	062, 15
FV	063, 15
f P/R	

Section 13: Straight-Line Depreciation

ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
÷	001, 10
1	002, 1
2	003, 2
=	004, 36
STO 1	005, 44 1
x≷y	006, 34
STO 2	007, 44 2
−	008, 30
1	009, 1
=	010, 36
STO 0	011, 44 0
1	012, 1
f SL	013, 42 23
X	014, 20
RCL 1	015, 45 1
=	016, 36
STO 3	017, 44 3
RCL PV	018, 45 13
−	019, 30
x≷y	020, 34
PV	021, 13
RCL n	022, 45 11

ALG KEYSTROKES	DISPLAY
−	023, 30
RCL 1	024, 45 1
n	025, 11
RCL 0	026, 45 0
g x=0	027, 43 35
g GTO 038	028,43,33,038
RCL 2	029, 45 2
g PSE	030, 43 31
RCL 0	031, 45 0
f SL	032, 42 23
R/S	033, 31
1	034, 1
STO + 0	035,44 40 0
STO + 2	036,44 40 2
g GTO 029	037,43,33,029
RCL 2	038, 45 2
g PSE	039, 43 31
RCL PV	040, 45 13
−	041, 30
RCL FV	042, 45 15
=	043, 36
RCL 3	044, 45 3
g GTO 033	045,43,33,033
f P/R	

Section 13: Declining-Balance Depreciation

ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
÷	001, 10
1	002, 1
2	003, 2
=	004, 36
STO 1	005, 44 1
x≷y	006, 34
STO 2	007, 44 2
−	008, 30
1	009, 1
=	010, 36
STO 0	011, 44 0
1	012, 1
f DB	013, 42 25
X	014, 20
RCL 1	015, 45 1
=	016, 36
STO 3	017, 44 3
RCL PV	018, 45 13
−	019, 30
x≷y	020, 34

ALG KEYSTROKES	DISPLAY
PV	021, 13
RCL 0	022, 45 0
g x=0	023, 43 35
g GTO 034	024,43,33,034
RCL 2	025, 45 2
g PSE	026, 43 31
RCL 0	027, 45 0
f DB	028, 42 25
R/S	029, 31
1	030, 1
STO + 0	031,44 40 0
STO + 2	032,44 40 2
g GTO 025	033,43,33,025
RCL 2	034, 45 2
g PSE	035, 43 31
RCL PV	036, 45 13
−	037, 30
RCL FV	038, 45 15
=	039, 36
RCL 3	040, 45 3
g GTO 029	041,43,33,029
f P/R	

Section 13: Sum-of-the-Years-Digits Depreciation

ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
÷	001, 10
1	002, 1
2	003, 2
=	004, 36
STO 1	005, 44 1
x≷y	006, 34
STO 2	007, 44 2
−	008, 30
1	009, 1
=	010, 36
STO 0	011, 44 0
1	012, 1
f SOYD	013, 42 24
X	014, 20
RCL 1	015, 45 1
=	016, 36
STO 3	017, 44 3
RCL PV	018, 45 13
−	019, 30
x≷y	020, 34
PV	021, 13
RCL n	022, 45 11

ALG KEYSTROKES	DISPLAY
−	023, 30
RCL 1	024, 45 1
n	025, 11
RCL 0	026, 45 0
g x=0	027, 43 35
g GTO 038	028,43,33,038
RCL 2	029, 45 2
g PSE	030, 43 31
RCL 0	031, 45 0
f SOYD	032, 42 24
R/S	033, 31
1	034, 1
STO + 0	035,44 40 0
STO + 2	036,44 40 2
g GTO 029	037,43,33,029
RCL 2	038, 45 2
g PSE	039, 43 31
RCL PV	040, 45 13
−	041, 30
RCL FV	042, 45 15
=	043, 36
RCL 3	044, 45 3
g GTO 033	045,43,33,033
f P/R	

Section 13: Full- and Partial- Year Depreciation with Crossover

ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
÷	001, 10
1	002, 1
2	003, 2
=	004, 36
STO 6	005, 44 6
RCL n	006, 45 11
−	007, 30
x≷y	008, 34
=	009, 36
STO 4	010, 44 4
R↓	011, 33
STO 0	012, 44 0
1	013, 1
STO − 0	014,44 30 0
STO 2	015, 44 2
STO 3	016, 44 3
f DB	017, 42 25
X	018, 20
RCL 6	019, 45 6
=	020, 36
STO 1	021, 44 1
RCL PV	022, 45 13
−	023, 30
x≷y	024, 34
PV	025, 13
RCL 1	026, 45 1
RCL PV	027, 45 13
−	028, 30
RCL FV	029, 45 15
=	030, 36
x≷y	031, 34
RCL 0	032, 45 0
1	033, 1
g x≤y	034, 43 34
g GTO 042	035,43,33,042
R↓	036, 33
R↓	037, 33
1	038, 1
g PSE	039, 43 31

ALG KEYSTROKES	DISPLAY
R↓	040, 33
R/S	041, 31
1	042, 1
STO + 2	043,44 40 2
STO − 0	044,44 30 0
f DB	045, 42 25
STO + 1	046,44 40 1
STO 5	047, 44 5
RCL PV	048, 45 13
−	049, 30
RCL FV	050, 45 15
÷	051, 10
RCL 4	052, 45 4
=	053, 36
g x≤y	054, 43 34
g GTO 057	055,43,33,057
g GTO 069	056,43,33,069
R↓	057, 33
0	058, 0
RCL 0	059, 45 0
g x≤y	060, 43 34
g GTO 090	061,43,33,090
RCL PV	062, 45 13
−	063, 30
RCL 5	064, 45 5
PV	065, 13
1	066, 1
STO − 4	067,44 30 4
g GTO 043	068,43,33,043
RCL 4	069, 45 4
n	070, 11
0	071, 0
STO 6	072, 44 6
1	073, 1
STO − 2	074,44 30 2
STO + 0	075,44 40 0
RCL 5	076, 45 5
STO − 1	077,44 30 1
RCL 3	078, 45 3
f SL	079, 42 23
STO + 1	080,44 40 1

Full- and partial-Year Depreciation with Crossover (continued)

ALG KEYSTROKES	DISPLAY
1	081, 1
STO − 0	082,44 30 0
STO + 2	083,44 40 2
STO + 3	084,44 40 3
R↓	085, 33
RCL 0	086, 45 0
1	087, 1
g x≤y	088, 43 34
g GTO 078	089,43,33,078
R↓	090, 33

ALG KEYSTROKES	DISPLAY
R↓	091, 33
RCL 2	092, 45 2
g PSE	093, 43 31
R↓	094, 33
R/S	095, 31
RCL 6	096, 45 6
g x=0	097, 43 35
g GTO 078	098,43,33,078
g GTO 062	099,43,33,062
f P/R	

Section 14: Lease with Advance Payments - Solving For Payment

ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
g END	001, 43 8
f CLEAR FIN	002, 42 34
RCL 0	003, 45 0
−	004, 30
RCL 1	005, 45 1
n	006, 11
RCL 2	007, 45 2
i	008, 12
1	009, 1

ALG KEYSTROKES	DISPLAY
CHS	010, 16
PMT	011, 14
PV	012, 13
+	013, 40
RCL 1	014, 45 1
÷	015, 10
RCL 3	016, 45 3
x≥y	017, 34
=	018, 36
f P/R	

Section 14: Lease with Advance Payments - Solving For Yield

ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
g END	001, 43 8
f CLEAR FIN	002, 42 34
RCL 0	003, 45 0
−	004, 30
RCL 1	005, 45 1
n	006, 11
RCL 2	007, 45 2

ALG KEYSTROKES	DISPLAY
PMT	008, 14
X	009, 20
RCL 1	010, 45 1
−	011, 30
RCL 3	012, 45 3
PV	013, 13
i	014, 12
RCL g 12÷	015,45,43 12
f P/R	

Section 14: Advance Payments With Residual - Solving for Payment

ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
g END	001, 43 8
f CLEAR FIN	002, 42 34
RCL 0	003, 45 0
n	004, 11
RCL 1	005, 45 1
i	006, 12
RCL 3	007, 45 3
FV	008, 15
PV	009, 13
+	010, 40
RCL 2	011, 45 2
=	012, 36
STO 5	013, 44 5
0	014, 0

ALG KEYSTROKES	DISPLAY
FV	015, 15
RCL n	016, 45 11
−	017, 30
RCL 4	018, 45 4
n	019, 11
1	020, 1
CHS	021, 16
PMT	022, 14
PV	023, 13
+	024, 40
RCL 4	025, 45 4
÷	026, 10
RCL 5	027, 45 5
x≷y	028, 34
=	029, 36
f P/R	

Section 15: Nominal Rate Converted to Effective Rate

ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
f CLEAR FIN	001, 42 34
n	002, 11
x≷y	003, 34
÷	004, 10
x≷y	005, 34
i	006, 12

ALG KEYSTROKES	DISPLAY
1	007, 1
PV	008, 13
FV	009, 15
1	010, 1
n	011, 11
i	012, 12
f P/R	

Section 16: 30/360 Day Basis Bonds

ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
f CLEAR FIN	001, 42 34
g BEG	002, 43 7
RCL 2	003, 45 2
÷	004, 10
2	005, 2
PMT	006, 14
STO 6	007, 44 6
+	008, 40
RCL 5	009, 45 5
FV	010, 15
RCL 3	011, 45 3
RCL 4	012, 45 4
g ΔDYS	013, 43 26
R↓	014, 33
÷	015, 10
1	016, 1
8	017, 8
0	018, 0
n	019, 11
g FRAC	020, 43 24
1	021, 1
−	022, 30
x≷y	023, 34
X	024, 20
RCL 6	025, 45 6

ALG KEYSTROKES	DISPLAY
=	026, 36
STO 6	027, 44 6
RCL 6	028, 45 6
RCL 0	029, 45 0
g x=0	030, 43 35
g GTO 041	031,43,33,041
÷	032, 10
2	033, 2
i	034, 12
PV	035, 13
CHS	036, 16
−	037, 30
x≷y	038, 34
=	039, 36
g GTO 000	040,43,33,000
R↓	041, 33
+	042, 40
RCL 1	043, 45 1
=	044, 36
CHS	045, 16
PV	046, 13
i	047, 12
X	048, 20
2	049, 2
=	050, 36
f P/R	

Section 16: Annual Coupon Bonds

ALG KEYSTROKES	DISPLAY
f P/R	
f CLEAR PRGM	000,
f CLEAR FIN	001, 42 34
g END	002, 43 8
RCL 0	003, 45 0
n	004, 11
RCL 2	005, 45 2
PMT	006, 14
RCL 1	007, 45 1
i	008, 12
RCL 3	009, 45 3
FV	010, 15
PV	011, 13
RCL 5	012, 45 5
−	013, 30
EEX	014, 26
6	015, 6
CHS	016, 16
=	017, 36
STO 6	018, 44 6
RCL 5	019, 45 5

ALG KEYSTROKES	DISPLAY
g ΔDYS	020, 43 26
STO 7	021, 44 7
RCL 6	022, 45 6
RCL 4	023, 45 4
g ΔDYS	024, 43 26
÷	025, 10
RCL 7	026, 45 7
n	027, 11
0	028, 0
PMT	029, 14
FV	030, 15
RCL n	031, 45 11
X	032, 20
RCL 2	033, 45 2
CHS	034, 16
+	035, 40
R/S	036, 31
x≷y	037, 34
=	038, 36
CHS	039, 16
f P/R	

Appendix B

Formulas Used

Real Estate

Wrap-Around Mortgage

n_1 = number of years remaining in original mortgage.

PMT_1 = yearly payment of original mortgage.

PV_1 = remaining balance of original mortgage.

n_2 = number of years in wrap-around mortgage.

PMT_2 = yearly payment of wrap-around mortgage.

PV_2 = total amount of wrap-around mortgage.

r = interest rate of wrap-around mortgage as a decimal.

FV = balloon payment.

$$PV_2 - PV_1 = \frac{PMT_2\left[1-(1+r)^{-n_2}\right]}{r} - \frac{PMT_1\left[1-(1+r)^{-n_1}\right]}{r} + FV(1+r)^{-n_2}$$

After-Tax Cash Flows

$ATCF_k$ = After-Tax Cash Flow for kth year.

Int_k = interest for kth year.

Dep_k = depreciation for kth year.

r = appropriate tax rate.

NOI = Net Operating Income.

$ATCF_k = NOI\ (1 - r) - 12 \times PMT + r \times (Int_k + Dep_k)$.

After-Tax Net Cash Proceeds of Resale

CO = capital purchase.

CPR = sales price − closing costs.

r = marginal tax rate.

c= capital gains tax rate.

$NCPR = CPR$ − remaining balance of mortgage.

$ATNCPR = NCPR - r \times$ (Total Dep. − SL Dep.) $- c \times (CPR - CO +$ SL Dep.)

$= NCPR - [c \times (CPR - CO) + r \times$ (Total Dep.) $+(c - r) \times$ (SL Dep.)]

Lending

Loans With a Constant Amount Paid Towards Principal

BAL_k = remaining balance after time period k.

$CPMT$ = Constant payment to principal.

BAL_k = PV − (k x $CPMT$)

kth payment to interest = i (BAL_k) = $(PMT_i)_k$

kth total payment = $CPMT$ + $(PMT_i)_k$

Add-On Interest Rate to APR

r = add-on rate as a decimal.

n = number of monthly payments.

APR = 1200i, where i is the solution in the following equation:

$$\frac{n}{1+\frac{n}{12}r} = \frac{1-(1+i)^{-n}}{i}$$

Add-On to APR with Credit Life

CL = credit life as decimal.

AMT = loan amount.

FC = finance charge.

$$\left[\frac{1+\left(\frac{n}{12}\right)r}{1-\left(\frac{n}{12}\right)\times CL-\left(\frac{n}{12}\right)^2\times CL\times r}\right]\times AMT = G$$

$$\frac{G}{n} = PMT$$

$$\frac{G\times CL\times n}{12} = \text{amount of credit life}$$

$$FC = (G - AMT - CL)$$

Rule of 78's Rebate

PV = finance charge.

I_k = interest charged at month k.

n = number of months in loan.

$$I_k = \frac{2(n-k+1)}{n(n+1)}PV$$

$$Rebate = \frac{(n-k)I_k}{2}$$

$$BAL_k = (n-k) \times PMT - Rebate_k$$

Graduated Payment Mortgage

n = total number of payments in the loan
I = interest rate per payment period, as a decimal
A = number of payments per year
B = number of years that payments increase
C = percentage increase in periodic payments (as a decimal)
PMT_1 = amount of the first payment
PV = amount of the loan.

$$PV = PMT_1 \left\{ \left[\frac{1-(1+I)^{-A}}{I} \right] \left[\frac{(1+Q)^B - 1}{Q} \right] + \frac{(1+C)^B \left[\frac{1-(1+I)^{-(n-AB)}}{I} \right]}{(1+I)^{AB}} \right\}$$

where:

$$Q = \frac{1+C}{(1+I)^A} - 1$$

Skipped Payments

A = number of payments per year.
B = number of years.
C = annual percentage rate as decimal.
D = periodic payment amount.
E = loan amount.
K = number of last payment before payments close the first time.
L = number of skipped payment.

$$D_{\text{END}} = \frac{E}{\left[1-\left(1+\frac{C}{A}\right)^{-AB} \right]} \times \frac{\left[\left(1+\frac{C}{A}\right)^{A} - 1 \right] \frac{C}{A}}{\left[\left(1+\frac{C}{A}\right)^{A} - \left(1+\frac{C}{A}\right)^{A-K} + \left(1+\frac{C}{A}\right)^{A-L-K} - 1 \right]}$$

$$D_{\text{BEGIN}} = \frac{D_{\text{END}}}{1+\frac{C}{A}}$$

Savings

Compounding Periods Different From Payment Periods

C = number of compounding periods per year.

P = number of payment periods per year.

i = periodic interest rate, expressed as a percentage.

$r = i / 100$, periodic interest rate expressed as a decimal.

$i_{PMT} = ((1 + r / C)^{C/P} - 1)100$

Investment Analysis

Lease vs. Purchase

PMT_p = loan payment for purchase.

PMT_L = lease payment.

I_n = interest portion of PMT_p for period n.

D_n = depreciation for period n.

M_n = maintenance for period n.

T = marginal tax rate.

$$\text{Net purchase advantage} = \sum_{n=1}^{k} \frac{\text{cost of leasing}(n) - \text{cost of owning}(n)}{(1+i)^n}$$

Cost of leasing(n) $= (1 - T)\, PMT_L$

Cost of owning(n) $= PMT_p - T(I_n + D_n) + (1 - T)M_n$

Break-Even Analysis and Operating Leverage

GP = Gross Profit.

P = Price per unit.

V = Variable costs per unit.

F = Fixed costs.

U = number of Units.

OL = Operating Leverage.

$GP = U(P - V) - F$

$$OL = \frac{U(P-V)}{U(P-V)-F}$$

Profit and Loss Analysis

Net income = (1 − tax) (net sales price − manufacturing expense − operating expense)

Net sales price = list price(1 − discount rate)

where operating expense represents a percentage of net sales price.

Securities and Options

Discounted Notes

Price (given discount rate)

B = number of days in year (annual basis).

DR = discount rate (as a decimal).

DSM = number of days from settlement date to maturity date.

P = dollar price per $100 per value.

RV = redemption value per $100 par value.

$$P = [RV] - \left[DR \times RV \times \frac{DSM}{B} \right]$$

Yield (given price)

B = number of days in year (annual basis).

DSM = number of days from settlement date to maturity date.

P = dollar price per $100 par value.

RV = redemption value per $100 par value.

Y = annual yield of investment with security held to maturity (as a decimal).

$$Y = \left[\frac{RV - P}{P} \right] \times \left[\frac{B}{DSM} \right]$$

Black-Scholes Formula for Valuing European Options

P = current asset price.

$r\%$= risk-free rate (continuous, per time unit).

$s\%$ = volatility (continuous, per time unit).

T = term of option (same time unit as $r\%$ and $s\%$).

X = exercise price of option.

$N(z)$= probability that a unit normal random variable is less than z.

$$Call\ Value = P \times N(d_1) - Q \times N(d_2)$$

$$Put\ Value = Call\ Value + Q - P$$

where:

$$d_1 = LN(P/Q)/v + v/2,\ d_2 = d_1 - v$$

$$Q = Xe^{(-T \times r\%/100)},\ v = s\%/100 \times \sqrt{T}$$

Forecasting

Simple Moving Average

$\bar{x}$ = moving average.

m = number of elements in moving average.

$$\bar{x}_1 = \frac{x_1 + x_2 + x_3 + \ldots + x_m}{m}$$

$$\bar{x}_2 = \frac{x_2 + x_3 + x_4 + \ldots + x_{m-1}}{m}$$

etc.

Seasonal Variation Factors Based on a Centered Moving Average

$\bar{x}_c$ = centered moving average

m = number of elements in the centered moving average.

$$\bar{x}_c = \frac{\frac{x_1}{2} + (x_2 + x_3 + \ldots + x_m) + \frac{x_{m+1}}{2}}{m}$$

SV = Seasonal variation factor.

x_i = value of the ith data point.

$\bar{x}_i$ = centered moving average of the ith data point.

$$SV = \frac{x_i}{\bar{x}_i}$$

Gompertz Curve Trend Analysis

$$y = ca^{(b^x)}$$

where x, y, a, b, and c are positive.

$$b = \left(\frac{S_3 - S_2}{S_2 - S_1}\right)^{1/n}$$

$$c = \exp\left[\frac{1}{n}\left(\frac{S_1 S_3 - {S_2}^2}{S_1 + S_3 - 2S_2}\right)\right]$$

$$a = \exp\left[\frac{(b-1)(S_2 - S_1)}{b(b^n - 1)^2}\right]$$

where S_1, S_2, and S_3 are:

$$S_1 = \sum_{i=1}^{n} \ln y_i = n \ln c + b(\ln a)\frac{b^n - 1}{b - 1}$$

$$S_2 = \sum_{i=n+1}^{2n} \ln y_i = n \ln c + b^{n+1}(\ln a)\frac{b^n - 1}{b - 1}$$

$$S_3 = \sum_{i=2n+1}^{3n} \ln y_i = n \ln c + b^{2n+1}(\ln a)\frac{b^n - 1}{b - 1}$$

a, *b* and *c* are determined by solving the three equations above simultaneously.

Forecasting With Exponential Smoothing

α = smoothing constant ($0 < \alpha < 1$)

X_t = actual current period usage

Smoothed average, $S_t = \alpha X_t + (1 - \alpha)S_{t-1}$

Change, $C_t = S_t - S_{t-1}$

Trend, $T_t = \alpha X_t + (1 - \alpha)T_{t-1}$

Current period expected usage, $D_t = S_t + \frac{(1 - \alpha)}{\alpha} T_t$

Forecast of next period expected usage, $\hat{D}_{t+1} = S_t + \left(\frac{1}{\alpha}\right) T_t$

Error, $e_t = \hat{D}_t - X_t$

Cumulative error $= \sum_{t=1}^{m} e_t^2$

Initial conditions : $S_{t-1} = X_{t-1}$ and $T_{t-1} = 0$

Pricing Calculations

Markup and Margin Calculations

Ma = margin(%).

Mu = markup(%).

S = selling price.

C = cost.

$$Ma = 100\frac{S - C}{S}$$

$$Mu = 100\frac{S - C}{C}$$

$$S = \frac{C}{1 - \frac{Ma}{100}}$$

$$S = C\left(1 + \frac{Mu}{100}\right)$$

$$C = S\left(1 - \frac{Ma}{100}\right)$$

$$C = \frac{S}{1 + \frac{Mu}{100}}$$

$$Ma = \frac{Mu}{1 + \frac{Mu}{100}}$$

$$Mu = \frac{Ma}{1 - \frac{Ma}{100}}$$

Calculations of List and Net Prices with Discounts

L = List price.

N = Net price.

D = Discount(%).

$$D' = 1 - \frac{D}{100}$$

$$L = \frac{N}{D'_1 \times D'_2 \times \ldots \times D'_x}$$

$$D_x = 100\left(1 - \frac{N}{L\left(D'_1 \times D'_2 \times \ldots \times D'_{x-1}\right)}\right)$$

Statistics

Exponential Curve Fit

$$y = Ae^{Bx}$$

$$\ln y = \ln A + Bx$$

$$B = \frac{\sum x_i \ln y_i - \frac{1}{n}\left(\sum x_i\right)\left(\sum \ln y_i\right)}{\sum {x_i}^2 - \frac{1}{n}\left(\sum x_i\right)^2}$$

$$A = \exp\left[\frac{\sum \ln y_i}{n} - B\frac{\sum x_i}{n}\right]$$

$$\hat{y} = Ae^{Bx}$$

Logarithmic Curve Fit

$$y = A + B(\ln x)$$

$$B = \frac{\sum y_i \ln x_i - \frac{1}{n}\left(\sum y_i\right)\left(\sum \ln x_i\right)}{\sum (\ln x_i)^2 - \frac{1}{n}\left(\sum \ln x_i\right)^2}$$

$$A = \frac{1}{n}\left(\sum y_i - B\sum \ln x_i\right)$$

$$\hat{y} = A + B(\ln x)$$

Power Curve Fit

$$y = Ax^B \qquad (A>0)$$

$$\ln y = \ln A + B(\ln x)$$

$$B = \frac{\sum (\ln x_i)(\ln y_i) - \frac{1}{n}\left(\sum \ln x_i\right)\left(\sum \ln y_i\right)}{\sum (\ln x_i)^2 - \frac{1}{n}\left(\sum \ln x_i\right)^2}$$

$$A = \exp\left[\frac{\sum \ln y_i}{n} - B\frac{\sum \ln x_i}{n}\right]$$

$$\hat{y} = Ax^B$$

Standard Error of the Mean

$$S_{\bar{x}} = \frac{S_x}{\sqrt{n}} \qquad S_{\bar{y}} = \frac{S_y}{\sqrt{n}}$$

Mean, Standard Deviation, Standard Error for Grouped Data

mean, $\bar{x} = \dfrac{\sum f_i x_i}{\sum f_i}$

standard deviation, $S_x = \sqrt{\dfrac{\sum f_i {x_i}^2 - \left(\sum f_i\right)\bar{x}^2}{\sum f_i - 1}}$

standard error, $S_{\bar{x}} = \dfrac{S_x}{\sqrt{\sum f_i}}$

Personal Finance

Tax-Free Retirement Account (IRA) or Keogh Plan

n = the number of years to retirement.

i = the compounded annual interest.

PMT = the total annual investment.

FV= future value, after applicable taxes.

r = the assumed tax rate on interest expressed as a decimal.

w = the withdrawal tax rate expressed as a decimal.

For ordinary taxable investment:

$$FV = \frac{PMT}{i(1-r)}\left[1+i(1-r)\right]\left\{\left[1+i(1-r)\right]^{n}-1\right\}$$

For tax-free investment:

$$FV = \frac{PMT \times (1-w)}{i}(1+i)\left[(1+i)^{n}-1\right]$$

Stock Portfolio Evaluation and Analysis

n = the number of issues held.

P_i = the current market price / share of a stock.

S_i = the number of shares of a stock held.

β_i = the beta coefficient of an individual stock.

T = the total present value of a portfolio.

Portfolio beta coefficient:

$$\beta = \sum_{i=1}^{n} \frac{P_i S_i \beta_i}{T}$$

Canadian Mortgages

r = annual interest rate expressed as a decimal.

$$\text{monthly factor} = \left[\left(1+\frac{r}{2}\right)^{1/6}-1\right]\times 100$$

Miscellaneous

Learning Curve for Manufacturing Cost

C_n = Cost of the nth unit, C_1 = Cost of the first unit.
n = number of units, r = learning factor.

$k = \ln r / \ln 2,\ C_n = C_1 n^k$

$\overline{C}_{ij}$ = the average cost of the ith through jth unit.

$$\overline{C}_{ij} = \frac{C_1}{j-i}\left[\frac{j^{k+1} - i^{k+1}}{k+1}\right]$$

This formula is only approximate and may give appreciable error at small i.

Queuing and Waiting Theory

n = number of servers.
λ = arrival rate of customers (Poisson input).
μ =service rate for each server (exponential service).
ρ = Intensity factor = λ / μ ($\rho < n$ for valid results).
P_0 = Probability that all servers are idle.
P_b = Probability that all servers are busy.
L_q = Average number of customers in queue.
L = Average number of customers in the system (waiting and being served).
T_q = Average waiting time in queue.
T = Average total time through the system.
$P(t)$ = Probability of waiting longer than time t.

$$P_0 = \left[\sum_{k=0}^{n-1}\frac{\rho^k}{k!} + \frac{\rho^n}{n!\left(1-\frac{\rho}{n}\right)}\right]^{-1}$$

$$P_b = \frac{\rho^n P_0}{n!\left(1-\frac{\rho}{n}\right)}$$

$$L_q = \frac{\rho P_b}{n-\rho}\ ,\quad L = L_q + \rho\ ,\quad T = L/\lambda\ ,\quad T_q = \frac{L_q}{\lambda}$$

$$P(t) = P_b e^{-(n\mu - \lambda)t}$$

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